VARYING FUNDAMENTAL CONSTANTS IN COSMOLOGY

by

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Abstract

We investigate the cosmological consequences of Bekenstein theory, in which the electric charge e takes on the value of a real scalar field. Cosmic string vortices in such theories are shown to act as source for variations in the electric charge, giving a significantly different value at the string core. The dielectric field arranges itself in the shape of a local string with a quantized magnetic flux presumably borrowing these features from the underlying Nielsen–Olesen vortex.

Furthermore we produce a self-consistent cosmological model from the Bekenstein theory. We show how this model can explain the recent evidence for a varying α, whilst still honouring constraints from fifth-force experiments. This is done by placing strong constraints upon the nature of the dark matter in the Universe. This cosmological model is investigated in detail, and it is shown, alongside Brans-Dicke theory, to motivate the formulation of novel anthropic considerations.

The link between non-minimally coupled scalar fields and weak equivalence principle violations can be shown to imply striking experimental differences between different varying-α theories. We suggest ways in which this can be used to distinguish observationally between varying e and varying c theories.

We also propose a self-consistent theory which combines our varying e theory with Brans-Dicke varying G theory. In this framework both α and G are allowed to vary simultaneously. The theory has similar behaviour to the constant G case, with G varying only a few percent through the history of the Universe.

Declaration

The work presented in this thesis was carried out in the Theoretical Physics Group at Imperial College under the supervision of Dr. João Magueijo. This thesis has not been submitted for a degree or diploma at any other university.

The material in chapter 2 has been published as "Nielsen–Olesen vortex in varying-α theories", J. Magueijo, H.B. Sandvik and T.W.B. Kibble, Phys. Rev. D64, 023521 (2001)

The main part of the work presented in chapter 3 has been published as "A simple varying-α cosmology", H.B. Sandvik, J.D. Barrow and J. Magueijo, Phys. Rev. Lett. 88, 031302 (2002). The remainder of the contents will be subject to future publication.

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For Mum, Dad, Endre and Nilottama

One field of work in which there has been too much speculation is cosmology. There are very few hard facts to go on, but theoretical workers have been busy constructing various models for the Universe, based on any assumptions that they fancy. These models are probably all wrong. It is usually assumed that the laws of Nature have always been the same as they are now. There is no justification for this. The laws may be changing, and in particular quantities which are considered to be constants of Nature may be varying with cosmological time. Such variations would completely upset the model makers.

P.A.M Dirac, On methods in theoretical physics, June 1968, Trieste

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Chapter 1

Introduction

The possibility that the constants of Nature may not be constant at all has long been entertained by physicists. Despite decades of enormous advances in fundamental particle physics we still have no idea why the constants of Nature take the values they do, let alone know the processes by which they acquire their values. Historically the gravitational constant has been the main subject to this sort of speculation. It started in 1935 when Milne set up his kinematic theory in which the gravitational constant is shown to grow $\propto t$[1, 2]. This was followed by Dirac's 1937 "Large Numbers Hypothesis"[3]. It states that the existence of certain large dimensionless numbers which arise in combinations of some cosmological numbers and physical constants was not a coincidence but a consequence of an underlying relationship between them.

To understand Dirac's concern about the "Large Numbers" we need to go into further detail[3]. Note first that the approximate number of nucleons in the universe is $N \approx \rho(cH^{-1})^3/m_p \sim 10^{80}$. Another dimensionless number is the ratio between the Hubble radius and the Compton wavelength of an electron which is $cH^{-1}/\frac{e^2}{m_e c^2} \sim 10^{40}$ thus approximately $\sqrt{N}$. A similar order of magnitude can be obtained if one considers the ratio between electromagnetic and gravitational forces on a hydrogen atom, namely $e^2/Gm_pm_e \sim 10^{40}$, again $\sim \sqrt{N}$. Dirac felt there had to be a deeper connection behind these seemingly coincidental numbers and proposed that this relationship should hold for all times not just the present day. Since the universe is expanding we know that the ratio between universal and atomic radii should go as $H^{-1} \propto t$. Thus if Dirac's hypothesis is correct we would expect $e^2/Gm_pm_e \propto t$ as well. Dirac went on to suggest that this dependence was carried by the gravitational constant and thus $G \propto t^{-1}$. Dirac's Large Number Hypothesis later attracted considerable interest[4, 5, 6, 7, 8]. Jordan[9, 10] was the first to consider how Dirac's hypothesis might be applied to forces other than gravity, however he rejected time variations in the weak interactions or the electron-proton mass ratio. Brans and Dicke[11] put G variation into a more rigorous theoretical framework within a scalar-tensor generalisation of General Relativity. This work was motivated in part by apparent discrepancies between observations and General Relativity in the solar system, but also by a wish to incorporate Mach's principle into General Relativity in the sense that the physical properties of space should have their origin in the matter contained therein. In this formalism variations in G were described self-consistently by the propagation of a scalar field which also acted as a source of space-time curvature. Teller (1948)[4] pointed out that $G \propto t^{-1}$, as followed from the LNH, would lead to earth surface temperatures near the boiling point of water. In 1967 attention shifted towards the electromagnetic fine structure constant $\alpha = e^2/\hbar c \approx 1/137.04$, when Gamow[12] in order to avoid these geological problems, suggested that the Large Number Hypothesis should be interpreted as a variation in the charge on the electron rather than in G so that $\alpha \propto t$. However this power-law variation in recent geological past was soon ruled out by other evidence.

The first proposition of a proper theory for a varying fine structure constant was made by Bekenstein in 1982[13]. He formulated a theory which could account for a variation in α or to be more precise, a variation in the fundamental charge, whilst still honouring important physical principles like causality and general covariance. This theory will form the basis of our work, and it will be described in more detail in a later chapter.

More recently, theories allowing for variations in fundamental constants arise from attempts to find a unified theory of quantum gravity. Most candidates like string and M-theories all feature extra dimensions, and in this scenario the 3+1 dimensional couplings we observe, are mere effective values of the true higher dimensional couplings and would thus vary as the size of the extra dimensions change. Various string theories exhibit different variations in the effective couplings, some of which allow both α and G to vary at the same time.

A more radical approach to varying α is seen in the so-called varying speed of light (VSL) theories. These theories attribute the change in the fine-structure constant to a varying light propagation speed[14, 15]. The motivation for such theories are their ability to solve the cosmological problems usually solved by inflation. Clearly the horizon problem is trivially solved by claiming a much higher light speed in the early universe, and under closer investigation these theories can be shown to also solve both flatness and monopole problems[14, 15, 16, 17, 18, 19]. VSL theories typically entail the breaking of Lorentz invariance, however there are exceptions (see e.g. [20] or [21]). Barrow and Magueijo developed a particular VSL theory in which the variation in the speed of light is coupled to the cosmological constant providing an explanation for the apparent smallness of the cosmological constant[22]. It is important to realise that speaking of variations in dimensionful constants is somewhat ambiguous, since a varying e theory can be reformulated as a VSL theory by an appropriate change of units (see e.g. the discussion in [15]). However the dynamics will be more transparent in the original frame, and it is in this context we can speak of a varying e vs. a varying c theory. Indeed we cannot experimentally distinguish one from the other as only dimensionless ratios can be measured[23].

The formulation and detailed investigation of varying-α cosmological theories have been further motivated by recent observations. The new observational many-multiplet technique of Webb et. al., [24], [25], exploits the extra sensitivity gained by studying relativistic transitions to different ground states using absorption lines in quasar (QSO) spectra at medium redshift. It has provided the first evidence that the fine structure constant might change with cosmological time[24, 25, 26]. The trend of these results is that the value of α was lower in the past, with $\Delta\alpha/\alpha = -0.72 \pm 0.18 \times 10^{-5}$ for $z \approx 0.5-3.5$. During the course of this thesis this data has been improved and the claim for a non-zero change in α with cosmological time is now at near 6σ level, so although independent confirmation is necessary, the results must now be taken seriously.

Another constraint on the time-variation in the fine structure constant comes from the natural reactor in Oklo, Gabon. By analysing nuclear and geochemical data one has been able to reconstruct the operating conditions of the reactor, and the thermal neutron capture cross sections of several nuclear species have been measured. Of particular interest is the ${}^{149}_{62}$Sm capture cross section

$$n +^{149}_{62} \mathrm{Sm} \rightarrow^{150}_{62} \mathrm{Sm} + \gamma \tag{1.1}$$

which was found by Shlyakhter in 1976[27] to be dominated by a narrow capture resonance of a neutron of energy near 0.1 eV. The existence of this resonance is a consequence of a near cancellation between the electromagnetic repulsive force and the strong interaction. By measuring this cross section at the time of reaction and relating it to the energy of the resonance one can translate the variation in this energy into a constraint on the time variation of the fine structure constant[27, 28]. The analysis of isotope ratios give $|\Delta\alpha/\alpha| \lesssim 10^{-7}$ over a period of 1.8 billion years (redshift $z \approx 0.1$)[27]. However a recent re-analysis of the Oklo bound using new Samarium samples provides two possible results for α[28],

$$\Delta\alpha/\alpha = -(0.7 \pm 0.9) \times 10^{-8} \tag{1.2}$$

or

$$\Delta\alpha/\alpha = (7.9 \pm 0.7) \times 10^{-8} \tag{1.3}$$

These two values correspond to two possible physical branches. Note that only the first solution is consistent with a null result, whereas the other solution has opposite sign to the Webb results.

There are also cosmological constraints on α coming from the Cosmic Microwave Background (CMB) and Big Bang Nucleosynthesis (BBN). The effects of varying fine structure constant on the CMB are twofold. Firstly it changes the temperature at which last scattering happens and secondly it changes the residual ionization after recombination. Both of these effects will affect the CMB anisotropies. For instance, an increase in α will lead to smaller sound horizon at the time of decoupling, with a corresponding shift of the C_l spectrum to higher multipoles. These effects have been used to set upper limits on $|\Delta\alpha/\alpha|$ of the order a few per cent[29, 30], consistent with a null result. The amount of ^{4}He produced during nucleosynthesis is mainly determined by the neutron to proton ratio at the freeze-out of weak interactions which is again directly dependent on the proton neutron mass difference. Electromagnetic energy contributes differently to neutron and proton masses[31], thus a change in the electromagnetic coupling will lead to a change in the neutron proton mass difference. The α dependence of Δm can be modelled by[32] $\Delta m \approx 2.05 - 0.76 \times (1 + \Delta\alpha/\alpha)$ MeV. Avelino et. al.[29] utilised this connection to derive the bound $\Delta\alpha/\alpha = (-7 \pm 9) \times 10^{-3}$, although other investigations have claimed slightly less restrictive constraints[32].

Should the observations of Webb et. al. be confirmed it will indeed be one of the biggest discoveries in physics over the last half century, and finding a reasonable theoretical explanation will be a great challenge. No doubt most of the physical constructions employed by physicists will have to be reexamined. Bekenstein's theory is perhaps the most conservative theory with which to interpret the new results, in the sense that it does not require giving up any truly fundamental principles, such as covariance and Lorentz invariance. Within the framework of this theory the vacuum is pervaded by a dielectric medium screening the electric charge. The properties of this dielectric medium are determined by the electromagnetic field itself, within the context of a dynamical Lagrangian theory. It is the purpose of this thesis to explore and develop further Bekenstein's theory.

In the next chapter as an introduction to the subject, we consider soliton solutions to Bekenstein's theory, for which the fine structure constant is allowed to vary due to the presence of a dielectric field pervading the vacuum. More specifically we investigate the effects of a varying α upon a complex scalar field with a $U(1)$ electromagnetic gauge symmetry subject to spontaneous symmetry breaking. We find vortex solutions to this theory similar to the Nielsen–Olesen vortex. Near the vortex core the electric charge is typically much larger than far away from the string, lending these strings a superconducting flavour. In general the dielectric field coats the usual local string with a global string envelope. We discuss the cosmological implications of networks of such strings, with particular emphasis on their ability to generate inhomogeneous recombination scenarios. We also consider the possibility of the dielectric being a charged free field. Even though the vacuum of such a field is trivial, we find that the dielectric arranges itself in the shape of a local string, with a quantized magnetic flux at the core — presumably borrowing these topological features from the underlying Nielsen–Olesen vortex.

In chapter 3 we explore the cosmological consequences of a simple theory in which the electric charge e is allowed to vary. The theory is locally gauge and Lorentz invariant, and satisfies general covariance. We find that in this theory, α remains almost constant in the radiation era, undergoes small increase in the matter era, but approaches a constant value when the universe starts accelerating due to the presence of a positive cosmological constant. This model satisfies geonuclear, nucleosynthesis, and CMB constraints on time-variation in α, while fitting simultaneously the observed accelerating universe and the recent high-redshift evidence for small α variations in quasar spectra. It also places specific restrictions on the nature of the dark matter. Further tests, involving stellar spectra and the Eötvös experiment, are proposed.

Chapter 4 investigates further and more deeply the behaviour of a time-varying fine structure 'constant' $\alpha(t)$ during the early and late phases of universes dominated by the kinetic energy of changing $\alpha(t)$, radiation, dust, curvature, and lambda, respectively. We show that after leaving an initial vacuum-dominated phase during which α increases, α remains constant in universes like our own during the radiation era, and then increases slowly, proportional to a logarithm of cosmic time, during the dust era. If the universe becomes dominated by negative curvature or a positive cosmological constant then α tends rapidly to a constant value. The effect of an early period of de Sitter or power-law inflation is to drive α to a constant value. Various cosmological consequences of these results are discussed with reference to recent observational studies of the value of α from quasar absorption spectra and to the existence of life in expanding universes.

In chapter 5 we study inhomogeneous cosmological variations in α in Friedmann universes. Inhomogeneous motions of the scalar field driving changes in α display spatial oscillations that decrease in amplitude with increasing time. The inhomogeneous evolution quickly approaches that found for exact Friedmann universes.

Chapter 6 contains a discussion of novel anthropic arguments, as we demonstrate how in some cosmological theories with varying constants there are anthropic reasons why the expansion of the universe must not be too *close* to flatness or the cosmological constant too close to zero. Using exact theories which incorporate time-variations in α and in G we show how the presence of negative spatial curvature and a positive cosmological constant play an essential role in bringing to an end variations in these 'constants' during any dust-dominated era of a universe's expansion. In spatially flat universes with $\Lambda = 0$ the fine structure constant grows to a value which makes the existence of atoms impossible.

The recent evidence for a time-varying fine structure 'constant' has raised an important question. Are the results to be interpreted as a varying e, c, $\hbar$, or a combination thereof? In chapter 7 we consider as examples a simple varying electric charge theory and a varying speed of light theory (VSL) and prove that for the same type of dark matter they predict opposite senses of variation in α over cosmological times. We also show that unlike varying e theories, some VSL theories do not predict violations of the weak equivalence principle (WEP). Varying e theories which explain astronomical inferences of varying α predict WEP violations only an order of magnitude smaller than existing Eötvös experiment limits but will be decisively tested by STEP. We finally exhibit a set of atomic-clock and related experiments for which *all* (hyperbolic) varying α theories predict non-null results. They provide independent tests of the recent astronomical evidence.

In chapter 8 we formulate a simple extension of general relativity which incorporates space-time variations in the Newtonian gravitation 'constant', G, and the fine structure 'constant', α, which generalises Brans-Dicke theory and our theory of varying α. We determine the behaviour of Friedmann universes in this theory. In the radiation and dust-dominated eras αG approaches a constant value and the rate of variation of α is equal to the magnitude of the rate of variation in G. The expansion dynamics of the universe are dominated by the variation of G but the variation of G has significant effects upon the time variation of α. Time variations in α are extinguished by the domination of the expansion by spatial curvature or quintessence fields, as in the case with no G variation.

We conclude with chapter 9 where we sum up the main findings of this thesis. We also address the difficulties in matching the QSO data with theory, and explore possible future directions that our work could take. Finally in appendix B we add some unpublished results describing the nature of charged black hole solutions in our theory.

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Chapter 2

Nielsen–Olesen vortex in varying-α theories

2.1 Introduction

As mentioned in the previous chapter, Bekenstein's theory is perhaps the most conservative theory with which to interpret the Webb results[1, 2]. In such dynamical Lagrangian theory there will not only be temporal variations in α but also spatial variations in the surroundings of an object with an electromagnetic energy component.

The application of this theory to cosmology is clouded by the issue of determining how much of the matter in the Universe will act as a source for this dielectric medium [3] (see however [4, 5] and the following chapters of this thesis). Clearly one needs to understand the microphysics underlying the cosmological fluid, in particular the nature of the dark matter, in order to set up a consistent cosmological model. In this chapter we first turn to a more concrete and simpler situation. We consider a complex scalar field with a $U(1)$ gauge electromagnetic symmetry spontaneously broken, which couples to a dielectric field in accordance with Bekenstein's prescription. We then consider topologically non-trivial solutions to this theory, the counterpart of the Nielsen–Olesen vortex [6]. In the standard theory such vortices have well localized concentrations of energy, along a stable string-like core. Furthermore this core constitutes a magnetic field flux tube. Hence the vortex acts as a source for the dielectric vacuum proposed in [3], leading to a varying α in the vicinity of the string.

The value of e in the string core is therefore much larger (smaller) than the asymptotic value e_0 (larger or smaller depending on parameter signs). If e becomes much larger we obtain a situation vaguely similar to the superconducting strings of Witten [7]. Indeed in some sense, infinite charge could amount to superconductivity. In Sec. 2.2 we set up the formalism, and study the asymptotics of our solution, and in Sec. 2.3 present the full numerical solution. A qualitative discussion of the implications of cosmological networks of such strings is presented in Sec. 2.4. Such strings would combine local and global string elements in their evolution and energy loss mechanisms, as well as in their gravitational interaction with the surrounding matter. More distinctly they would generate inhomogeneities in the value of e, leading, among other things, to inhomogeneous recombination scenarios.

Another interesting connection, spelled out in Sec. 2.5, is the similarity between our solutions and fast-tracks, a construction found in VSL theories [8]. Such solitonic solutions to VSL allow for fast travel without a time-dilation effect. We discuss how the situation is distinctly different in the case of these strings — they still allow for fast travel in some sense, however they would induce a time-dilation effect of their own which has nothing to do with the special relativity effect.

The solution derived in Sec. 2.2 is but the simplest of many similar constructions involving solitonic solutions coupled to varying charge theories. In all of these a gauge field undergoing spontaneous symmetry breaking supplies a solitonic solution which acts as a source for a dielectric field. As a result a dielectric coating is superposed on the soliton, forcing the gauge coupling (or charge) to vary in the soliton core or near its vicinity. In Sec. 2.6 we discuss the possibility of gauging the dielectric field itself. In a concluding discussion, in Sec. 2.7 we also consider the possibility of non-Abelian gauge groups, and similar constructions with the morphology of monopoles and textures.

2.2 The model

We first describe Bekenstein's theory in the context of a charged complex scalar field undergoing spontaneous symmetry breaking. Let ϕ be a complex scalar field with a gauged $U(1)$ symmetry, and A_μ be the gauge field. Let the electric charge e be a variable, with $\epsilon = e/e_0$ where e_0 is some fixed electric charge. Under a gauge transformation $\delta\phi = i\Lambda\phi$, where Λ is a scalar function, one should impose $\delta A_\mu = -(\partial_\mu \Lambda)/e$, so that the derivative $D_\mu = \partial_\mu + ieA_\mu$ transforms covariantly. The gauge invariant electromagnetic field tensor is then

$$F_{\mu\nu} = \frac{1}{\epsilon}(\partial_\mu(\epsilon A_\nu) - \partial_\nu(\epsilon A_\mu)) \tag{2.1}$$

and the Lagrangian is:

$$\mathcal{L} = -(D^\mu\phi)^\star D_\mu\phi - V(\phi) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{\omega}{2\epsilon^2}\partial_\mu\epsilon\partial^\mu\epsilon \tag{2.2}$$

(we are using a metric with signature $- + ++$ as we will do consistently throughout this thesis). The first three terms constitute the matter Lagrangian, while the last term governs the dynamics of ϵ. We adopt the proverbial Mexican hat potential:

$$V(\phi) = m^2|\phi|^2 + \lambda|\phi|^4, \tag{2.3}$$

with λ and $m^2 < 0$ fixed parameters. The vacuum is then the circle $|\phi| = \phi_0 = \sqrt{-m^2/(2\lambda)}$.

We first introduce a transformation which simplifies Bekenstein's theory enormously. We note that by defining an auxiliary gauge potential $a_\mu = \epsilon A_\mu$ and field tensor:

$$f_{\mu\nu} = \epsilon F_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu \tag{2.4}$$

the Lagrangian becomes

$$\mathcal{L} = -(D^\mu\phi)^\star D_\mu\phi - V(\phi) - \frac{1}{4}\frac{f_{\mu\nu}f^{\mu\nu}}{\epsilon^2} - \frac{\omega}{2\epsilon^2}\partial_\mu\epsilon\partial^\mu\epsilon, \tag{2.5}$$

in which $D_\mu = \partial_\mu + ie_0 a_\mu$. Hence we have eliminated the dependence on ϵ in the matter Lagrangian apart from dividing the f^2 term by ϵ^2. This greatly simplifies the variational problem regardless of which variables we decide to label as physical (which is essentially a matter of convention). Indeed zero variation with respect to $\{\phi, A_\mu, \epsilon\}$ is equivalent to zero variation with respect to $\{\phi, a_\mu, \epsilon\}$. We have also exposed an interesting similarity between this theory and Brans–Dicke changing-G theory. In the latter one multiplies the Ricci scalar (essentially a f^2 term) by a scalar field, which also does not appear explicitly elsewhere (other than in its own kinetic terms or potential).

Variation of (2.5) with respect to ϕ produces the equation:

$$D_\mu D^\mu \phi = \frac{\partial V}{\partial \phi^\star}, \tag{2.6}$$

in which we may use $D_\mu = \partial_\mu + ie_0 a_\mu$ or $D_\mu = \partial_\mu + ieA_\mu$. Variation with respect to a_μ now produces straightforwardly:

$$\partial_\mu \frac{f^{\mu\nu}}{\epsilon^2} = \partial_\mu \frac{F^{\mu\nu}}{\epsilon} = j^\nu = ie_0[\phi^\star D^\nu \phi - \phi(D^\nu \phi)^\star] \tag{2.7}$$

The first pair of Maxwell equations need to be modified accordingly. By use of the antisymmetric properties of the electromagnetic field tensor, $f_{\mu\nu}$, we can show that the following relation is satisfied

$$\frac{\partial f_{\alpha\beta}}{\partial x^\gamma} + \frac{\partial f_{\beta\gamma}}{\partial x^\alpha} + \frac{\partial f_{\gamma\alpha}}{\partial x^\beta} = \frac{\partial \epsilon F_{\alpha\beta}}{\partial x^\gamma} + \frac{\partial \epsilon F_{\beta\gamma}}{\partial x^\alpha} + \frac{\partial \epsilon F_{\gamma\alpha}}{\partial x^\beta} = 0 \tag{2.8}$$

Finally variation with respect to ϵ leads to:

$$\Box \ln \epsilon = -\frac{1}{2\omega}\frac{f^2}{\epsilon^2} = -\frac{1}{2\omega}F^2. \tag{2.9}$$

These equations, in the $\{A_\mu, F_{\mu\nu}\}$ representation, are nothing but Bekenstein's equations. However the transformation we have used has simplified the derivation greatly, and it will also simplify the search for solutions in what follows.

We now seek solutions similar to the Nielsen–Olesen vortex in this theory. We therefore introduce the ansatz $\phi = \chi(r)e^{in\theta}$ and $a_\theta = a(r)$ with all other components for a_μ set to zero. We define a magnetic field out of the $f_{\mu\nu}$ tensor, so that $\mathbf{b} = \nabla \times \mathbf{a}$. Hence the magnetic field is aligned with the z direction and has value:

$$b = b_z = \frac{1}{r}\frac{d}{dr}[ra]. \tag{2.10}$$

The dynamical equations are:

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{d\chi}{dr}\right) - \left[\left(\frac{n}{r} - e_0 a\right)^2 + m^2 + 2\lambda\chi^2\right]\chi = 0, \tag{2.11}$$

which is unmodified, and

$$\frac{d}{dr}\left(\frac{1}{r\epsilon^2}\frac{d}{dr}(ra)\right) + 2e_0\left(\frac{n}{r} - e_0 a\right)\chi^2 = 0 \tag{2.12}$$

and

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{d\ln\epsilon}{dr}\right) = -\frac{1}{2\omega}\frac{b^2}{\epsilon^2}. \tag{2.13}$$

To investigate the asymptotic behaviour far from the core, first recall that the scalar field takes on the constant value $\chi = \sqrt{-\frac{m^2}{2\lambda}}$ for $r \to \infty$. From Eq. (2.11) we then see that

$$a = \frac{n}{re_0}, \tag{2.14}$$

which also agrees with Eq. 2.12. From this we deduce that the fiux of b is quantized:

$$\int \mathbf{b} \cdot \mathbf{dS} = \oint \mathbf{a} \cdot \mathbf{dl} = \frac{2\pi n}{e_0} \tag{2.15}$$

but not the flux of $B = b/\epsilon$ in which ϵ takes the role of a magnetic permeability. We also find the asymptotic solution for ϵ:

$$\epsilon = \left(\frac{r}{r_0}\right)^{\frac{-I}{\omega\pi}}, \tag{2.16}$$

where I is the integral of $\mathbf{B}^2$ over a string cross section. Hence ϵ can only either go to zero (if $\omega > 0$) or infinity (if $\omega < 0$) far away from the string — corresponding to a logarithmic divergence in ψ; not surprising since (2.13) has sources at spatial infinity (at $z = \pm\infty$). Furthermore the energy in the ϵ field diverges away from the string. These

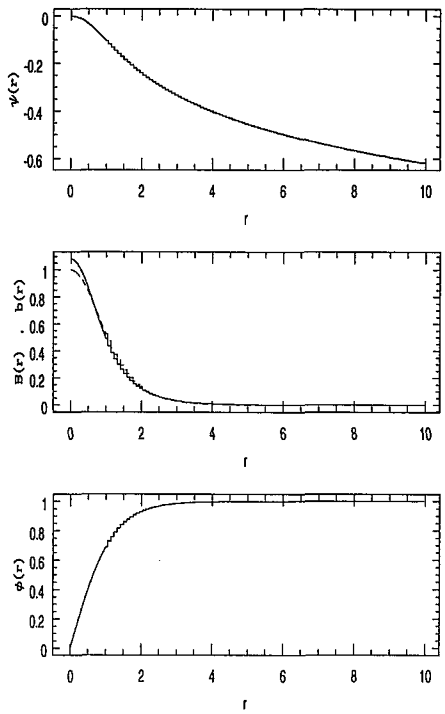

Figure 2.1: Numerical integration of the Nielsen–Olesen vortex. The first graph shows the solution of $\psi = ln\epsilon$ as a function of r, in Bekenstein's theory. The middle graph shows a plot of B with a constant electric charge (dashed line) compared to a plot of b in Bekenstein's theory. The third graph compares the solution of the ϕ field in the two theories; the solutions are indistinguishable.

features are well known properties of global strings. Indeed we have a local or gauged string (in the field ϕ) superposed on a global string (in the field ϵ).

In a cosmological setting these seemingly pathological divergences are naturally removed by the scale of curvature of the strings. Then the difference between the asymptotic and core values of the electric charge is roughly of the order of $(r_c/r_0)^{-I/(\omega\pi)}$, where r_c and r_0 are the curvature and core radii of the string respectively.

If we require that ψ has a positive definite energy, then $\omega > 0$ in which case the charge at the core should be much higher than its asymptotic value. It is in this case that we can claim a similarity between our construction and superconducting strings [7]. In some loose sense a diverging electric charge should be equivalent to superconductivity. Indeed applying an electric field upon a conductor in the interior of this string subjects the charge carriers to a force proportional to e. Hence the electric force applied to them is much larger than normal. If the resistance to which they are subject does not change, we can then ignore it — and it is in this sense that these strings could maintain persistent currents and therefore be labelled superconducting. Note that this is just a loose analogy: effects such as the expulsion of magnetic fields from the interior of the string are not present in this case.

Our solution also has vague similarities to the dilatonic string of [9], for which the string mass per unit length is a function of a scalar field.

2.3 Numerical Solutions of the Model

Equations (2.11)–(2.13), together with the asymptotic values at $r = 0$ and $r = r_c$, constitute a boundary value problem. For the sake of numerically solving this problem, it is convenient to make the following change of variables so as to avoid singularities:

$$a \rightarrow v = ar, \tag{2.17}$$

$$\epsilon \rightarrow \psi = \ln \epsilon. \tag{2.18}$$

We also reduce the problem to first order by introducing the new variables

$$\sigma = \frac{d\chi}{dr}, \qquad \eta = \frac{d\psi}{dr}. \tag{2.19}$$

The new set of equations suitable for numerical implementation is

$$\begin{aligned}
\frac{d\chi}{dr} &= \sigma, && (2.20)\\
\frac{d\sigma}{dr} &= -\frac{\sigma}{r} + \left(\left(\frac{n - e_0 v}{r} \right)^2 + m^2 + 2\lambda\chi^2 \right) \chi, && (2.21)\\
\frac{dv}{dr} &= br, && (2.22)\\
\frac{db}{dr} &= 2\eta b - \left(2e_0 \frac{n - e_0 v}{r} \chi^2 \right) e^{2\psi}, && (2.23)\\
\frac{d\psi}{dr} &= \eta, && (2.24)\\
\frac{d\eta}{dr} &= -\frac{\eta}{r} - \frac{1}{2\omega} b^2 e^{-2\psi}. && (2.25)
\end{aligned}$$

We first check our code on the Nielsen–Olesen vortex with constant ϵ. The results for the scalar field and the magnetic field compare well with the original work[6], and are shown as the dashed lines in Figure 2.1. We then allow the ϵ to vary, and incorporate Equations (2.20)–(2.25). The results are consistent with the asymptotic behaviour found in Equations (2.14)–(2.16), and are shown as the solid lines in Figure 2.1.

2.4 Qualitative discussion of a cosmological network

Varying-α strings, if formed at phase transitions, would have a complex evolution. It is conceivable that the string core would still be governed by the Nambu–Goto action. Also, presumably these strings, when crossing, would still intercommute (although this fact should be checked by numerical simulations). However, in addition to intercommuting, the dielectric field would act as a long-range force between the strings, creating a double mechanism for string interaction. Hence we should have something like a local string network, acting as a source for a global string network, the two networks being driven by their usual interaction mechanisms plus a complex interaction between the two.

Energy loss processes would again combine local and global string elements. The core string should develop small scale structure, via intercommutation, thereby emitting gravitational radiation. On the other hand the dielectric field would now supply a channel for the string to lose energy via the emission of scalar radiation. A combination of processes peculiar to local and global strings should therefore push these strings towards a scaling solution, but clearly we may expect such scaling solutions to be distinctly different from the usual ones.

The interactions of these strings and the other matter in the universe would also be rather peculiar. Gravitationally we would find a combination of the effects of local strings (and their conical flat space) and the more complex global string gravitational fields. However, predicting the density fluctuations in this scenario as a simple superposition of global and local fluctuations (that is the total spectrum as a weighted average of the separate spectra) is clearly too gross an approximation. The local and global string networks will be highly correlated, and have a strong feedback effect upon each other. Whatever the gravitational effects of these strings upon the surrounding matter will be, they have to be determined by simulations along the lines of [10, 11] specifically applied to varying-α string networks. Note that unlike conventional super-conducting cosmic strings we would not expect the equation of state of these strings to differ from standard ones.

However what would truly distinguish these strings, and their possible cosmological effects, is the fact that the electric charge varies in their vicinity. For a straight string the electron charge variation away from the core would be given by:

$$\frac{\Delta e}{e} = \frac{\beta}{\omega\pi} G\mu \ln \frac{r}{r_0}, \tag{2.26}$$

in which for the I defined after equation 2.16, we used $I = \beta G\mu$. Here μ is the string mass per unit length, and β is the fraction of this mass in the magnetic field flux tube. These fluctuations are therefore of the order of the gravitational potential induced by the strings.

Hence, in addition to acting as gravitational seeds for structure formation, these strings would affect any electromagnetic processes in their neighbourhood. A topical example is recombination. In the vicinity of these strings the hydrogen binding energy would suffer spatial variations, leading to inhomogeneous recombination. The implications of a homogeneous changing-α upon recombination and CMB anisotropy were studied in [12, 13]. A network of changing-α strings would provide additional effects.

2.5 A comparison of varying-α strings and fast tracks

Although there is a parallel between the solutions found in this chapter and fast-tracks [8], there is a crucial difference. Fast-tracks are string-like solutions to some covariant VSL theories such that the speed of light is much higher near the string core. Hence observers may move along the string core much faster than the asymptotic value of $c = c_\infty$. Moreover such "super-c_∞" speeds need not be relativistic, that is, they may still be much smaller than the local value of c, so that such observers would not be subject to time-dilation effects. Fast-tracks are what space travel is begging for: fast, "superluminal" travel, free from time dilation. It can be shown that a change of units transforms fast-tracks into wormholes.

In the case of our strings the situation is rather different. As α changes near the string core so will change the time rates associated with all electromagnetic processes. In particular an atomic clock, ticking to a rate $\tau \propto 1/\alpha^2$, will tick differently. Biological processes, being electromagnetic, also tick to this rate. If the charge decreases towards the string core, then alpha is smaller, and so the time scales τ of all electromagnetic processes increase. Unfortunately this situation is realized in the case $\omega < 0$, for which the dielectric energy density is not positive definite. Nonetheless let us consider further this case.

Using coordinate time we know that "super-c_∞" speeds cannot be achieved near the string. However, since $e \to 0$, it would then be possible to "pickle" observers moving along the string, since $\tau \to \infty$. If we were to measure speeds along the string in atomic clock units, then we could indeed measure "super-c_∞": the point is that natural organisms would be able to travel very large distances within their perceived time scales.

However the use of such strings for space travel would still lead to twin paradox effects: clearly a round trip would cause a large difference in ages between sedentary and nomadic twins. Curiously enough such a time dilation effect has nothing to do with relativistic speeds. It is simply given by

$$\Delta t = \Delta t_0 \oint \frac{dt}{\alpha^2}. \tag{2.27}$$

In this respect the varying-α strings considered here are distrinctly different from VSL fast-tracks.

2.6 Gauged dielectric field

We have noted that the dielectric coating surrounding our modified Nielsen–Olesen vortex is like a global string superposed on the usual gauged string (made up of charged scalar field and a magnetic flux tube). Further symmetry would be enforced if the dielectric itself were charged, that is if we replaced Bekenstein's real scalar field $\psi = \ln \epsilon$, by a complex field ψ, such that $\epsilon = e^{|\psi|}$. Upon gauging the $U(1)$ symmetry associated with ψ we therefore arrive at the Lagrangian:

$$\begin{aligned}\mathcal{L} &= -(D^\mu\phi)^\star D_\mu\phi - V(\phi) - \frac{1}{4}\frac{f_{\mu\nu}f^{\mu\nu}}{\epsilon^2} \\ &\quad -\omega[(\tilde{D}_\mu\psi)^\star(\tilde{D}^\mu\psi) + \frac{1}{4}G_{\mu\nu}G^{\mu\nu}],\end{aligned} \tag{2.28}$$

in which $\tilde{D}_\mu = \partial_\mu + igB_\mu$, where g the charge of the dielectric field, B^μ is the photon associated with the dielectric, and $G_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$ the corresponding field tensor. The equations for the scalar field ϕ and the gauge field a_μ remain unaffected, but the equation for ψ is now:

$$\tilde{D}_\mu \tilde{D}^\mu \psi = -\frac{1}{4\omega} f^2 e^{-2|\psi|} \frac{\psi}{|\psi|} \tag{2.29}$$

and for B_μ

$$\partial_\nu G^{\nu\mu} = j^\mu_\psi = ig[\psi^\star \tilde{D}^\mu \psi - \psi(\tilde{D}^\mu \psi)^\star]. \tag{2.30}$$

Under cylindrical symmetry these equations may be solved using the same ansatz for ϕ and a_μ as before, and the counterparts for the dielectric: $\psi = \xi(r)e^{im\theta}$, and $B_\theta = B(r)$ (with all other components of B_μ set to zero). The new equations are

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{d\xi}{dr}\right) - \left(\frac{m}{r} - gB\right)^2 \xi + \frac{f^2 e^{-2\xi}}{4\omega} = 0, \tag{2.31}$$

$$\frac{d}{dr}\left(\frac{1}{r}\frac{d}{dr}(rB)\right) + 2g\left(\frac{m}{r} - gB\right)\xi^2 = 0. \tag{2.32}$$

Given that f^2 is confined to the (local) string core, the asymptotics for the new fields are similar. While we still have $\chi = \sqrt{-\frac{m^2}{2\lambda}}$ and $a = n/(re_0)$, for $r \to \infty$, we find that ξ may go to any constant (if e_0 is the asymptotic value of the electric charge, $\xi \to 0$), while

$$B = \frac{m}{rg}. \tag{2.33}$$

Overall we find that the dielectric behaves like a local string superposed on the usual Nielsen–Olesen vortex. Its magnetic flux, associated with the gauge field B_μ, is quantized, with a quantum number m. This is particularly curious as there is nothing topological in the nature of the dielectric string. Somehow it borrows these features from the topological nature of the ϕ-string sourcing it. The ψ field does not have a potential, only kinetic terms plus a source at the string. Thus the ψ field can take on any covariantly constant value far away from the string, which amounts to constant $|\psi|$, and a phase equal to $m\theta$.

Notice that none of the points made in Section 2.4, regarding the cosmological implications of local strings coupled to an ungauged dielectric, apply to the strings considered in this section. The variations in e experienced in the surroundings of these strings are confined to microphysical distances, and have no direct cosmological implications.

It could also happen that ϕ and ψ are coupled to the same $U(1)$ gauge field. Then

$$\mathcal{L} = -(D^\mu\phi)^\star D_\mu\phi - V(\phi) - \frac{f_{\mu\nu}f^{\mu\nu}}{4\epsilon^2} - \omega(D_\mu\psi)^\star(D^\mu\psi), \tag{2.34}$$

leading to equations:

$$D_\mu D^\mu \phi = \frac{\partial V}{\partial \phi^\star}, \tag{2.35}$$

$$D_\mu D^\mu \psi = -\frac{1}{4\omega} f^2 e^{-2|\psi|} \frac{\psi}{|\psi|}, \tag{2.36}$$

and the gauge-field equation:

$$\partial_\nu \frac{f^{\mu\nu}}{\epsilon^2} = j^\mu_\phi + j^\mu_\psi, \tag{2.37}$$

$$j^\mu_\phi = ie_0[\phi^\star D^\nu\phi - \phi(D^\nu\phi)^\star], \tag{2.38}$$

$$j^\mu_\psi = ie_0[\psi^\star D^\nu\psi - \psi(D^\nu\psi)^\star]. \tag{2.39}$$

Studying the asymptotics of these equations we find that in this case the quantum number m associated with ψ would have to be the same as n. Indeed we have that $\phi = \chi(r)e^{in\theta}$, with $\chi = \sqrt{-\frac{m^2}{2\lambda}}$, and $a = n/(re_0)$; but now we should also have $\psi = \xi(r)e^{in\theta}$ with $\xi(r)$ going to any constant. More generally it could be that the charge of the ψ field is $g = ke$, where k is an integer (or more generally a rational number), in which case $m = kn$.

2.7 Concluding remarks

In this chapter we studied the counterpart of the Nielsen–Olesen vortex in Bekenstein's varying-α theory, by means of analytical asymptotic methods, and numerically. We found that such strings are covered by the dielectric medium characterizing Bekenstein's theory. This coating, in effect, looks like a global string superposed upon the local string core. The electric charge would thus vary (typically increase) as the string core is approached.

We then discussed possible cosmological implications of such strings. Clearly their networks will be much more complex than just the superposition of a local and a global string network. We pointed out the main aspects in which their dynamics and energy loss mechanisms will be more complex. Structure formation in these theories will also have more to it than just a superposition of results known to be true for the two types of network. In addition we highlighted a peculiar feature of these networks: their ability to generate inhomogeneities in the electric charge, and consequently (among other things) to generate inhomogeneous reionization scenarios.

In a brief section we compared these strings with fast-tracks: solitonic solutions to VSL theories along which fast travel without time-dilation effects is possible. We showed that while in some sense fast travel along these strings is possible, in those cases one cannot evade a time-dilation effect. Curiously enough this time dilation effect is present even if observers do not exceed non-relativistic speeds. It is an effect merely due to the fact that the pace of atomic clocks depends upon α, and slows down accordingly near the string core.

Finally we initiated an exploration of other solitonic solutions in these theories. We considered the possibility that the dielectric field itself might be a gauged. We found that in such a case, even if the dielectric is not endowed with a potential, it acquires topological features, e.g. quantization of its associated magnetic field flux in the string core.

Although in this work we restricted ourselves to gauged $U(1)$ symmetries it is possible to generalize our constructions to non-abelian symmetry groups. Indeed counterparts to Bekenstein's theory associated with strong interactions were discussed in [14]. Following such generalizations it would be possible to construct monopoles and textures (associated with $O(3)$ and $SU(2)$ gauge symmetries), covered by similar dielectric coatings. The only type of soliton which apparently could not be associated with changing-charge theories are domain walls, for which there is no associated gauge symmetry. We defer to future work the scrutiny of these more complicated solitons.

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Chapter 3

A Simple Varying-alpha Cosmology

3.1 Introduction

As mentioned in chapter 1, the observations by Webb and collaborators have provided the first evidence that the fine structure constant might change with cosmological time[1, 2, 3]. The trend of these results is that the value of α was lower in the past, with $\Delta\alpha/\alpha = -0.72 \pm 0.18 \times 10^{-5}$ for $z \approx 0.5 - 3.5$.

Another remarkable set of recent observations is of Type la supernovae in distant galaxies. These data have extended the Hubble diagram to redshifts, $z \geq 1$[4]. They imply an accelerated expansion of the universe. When combined with CMB data, the supernovae observations favour a flat universe with approximate matter density, $\Omega_m \approx 0.3$ and vacuum energy density, $\Omega_\Lambda \approx 0.7$. Studies have attempted to determine whether quintessential scalar fields could explain both cosmological dark matter and the recent acceleration of the universe, [5, 6, 7, 8, 9].

We will not here attempt to explain the acceleration of the universe. Instead, we show that by applying a generalisation of Bekenstein's varying-e theory in a cosmological setting including the cosmological constant, Λ, we are able to explain the magnitude and sense of the observed change in α. The main assumption is that the cold dark matter has magnetic fields dominating their electric fields. The magnetostatic energy then drives changes in α in the matter dominated epoch, but as the Universe starts to accelerate these changes become friction dominated and come to a halt. This gives a decelerated rate of change in α, just as the universe starts to accelerate, in accord with both data sets. The only energy scale we introduce is similar to the Planck scale, which also makes our model attractive. This model may be seen as a more conservative alternative to [10, 11], where a VSL scenario was proposed which could explain the observed acceleration of the universe and variations in α, as well as their remarkable coincidence in redshift space.

3.2 A scalar theory of varying α

Bekenstein's original theory[12] takes c and $\hbar$ to be constants and attributes variations in α to changes in e, or the permittivity of free space. In chapter 2 we introduced this theory in the context of a complex scalar field. Here, we will develop the relevant equations and formalisms in a cosmological setting. As seen in the previous chapter, the variation in α is achieved by letting e take on the value of a real scalar field which varies in space and time $e_0 \to e = e_0\epsilon(x^\mu)$, where ϵ is a dimensionless scalar field and e_0 is a constant denoting the present value of e. This means some well established assumptions, like charge conservation, must give way [13]. Still, the principles of local gauge invariance and causality are maintained, as is the scale invariance of the ϵ field.

Since e is the electromagnetic coupling, the ϵ field couples to the gauge field as ϵA_μ in the Lagrangian and the gauge transformation which leaves the action invariant is $\epsilon A_\mu \to \epsilon A_\mu + \chi_{,\mu}$, rather than the usual $A_\mu \to A_\mu + \chi_{,\mu}$. The gauge-invariant electromagnetic field tensor is again, $F_{\mu\nu} = \left((\epsilon A_\nu)_{,\mu} - (\epsilon A_\mu)_{,\nu}\right)/\epsilon$, which reduces to the usual form when ϵ is constant. The electromagnetic Lagrangian is still $\mathcal{L}_{em} = -F^{\mu\nu}F_{\mu\nu}/4$ and the dynamics of the ϵ field are controlled by the kinetic term $\mathcal{L}_\epsilon = -\frac{1}{2}\omega(\epsilon_{,\mu}\epsilon^{,\mu})/\epsilon^2$, as in [12] (we again use a metric with signature -+++). Here the coupling constant ω can be written as $\frac{\hbar c}{l^2}$, where l is the characteristic length scale of the theory, introduced for dimensional reasons. This constant length scale gives the scale down to which the electric field around a point charge is accurately Coulombic.

The corresponding energy scale, $\hbar c/l$, has to lie above a few tens of MeV to avoid conflict with experiment.

3.3 Cosmology in a varying α theory

We now go on to consider Bekenstein's theory in the cosmological setting suggested by the recent supernovae and CMB results. To simplify calculations, we invoke the transformation introduced in chapter 2. By defining an auxiliary gauge potential $a_\mu = \epsilon A_\mu$, and field tensor $f_{\mu\nu} = \epsilon F_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu$, the covariant derivative takes the usual form, $D_\mu = \partial_\mu + ie_0 a_\mu$. The dependence on ϵ in the Lagrangian then occurs only in the kinetic term for ϵ and in the $F^2 = f^2/\epsilon^2$ term. To simplify further we change variable: $\epsilon \to \psi \equiv ln\epsilon$. The total action becomes

$$S = \int d^4x\sqrt{-g}\left(\mathcal{L}_g + \mathcal{L}_{mat} + \mathcal{L}_\psi + \mathcal{L}_{em}e^{-2\psi}\right), \tag{3.1}$$

where $\mathcal{L}_\psi = -\frac{\omega}{2}\partial_\mu\psi\partial^\mu\psi$ and $\mathcal{L}_{em} = -\frac{1}{4}f_{\mu\nu}f^{\mu\nu}$. The gravitational Lagrangian is the usual $\mathcal{L}_g = \frac{1}{16\pi G}R$, with R the curvature scalar. Our theory generalises Bekenstein's approach by including the effects of the varying ϵ (or ψ) field on the gravitational dynamics of the expanding universe. The scalar field ψ plays a similar role to the dilaton in the low-energy limit of string and M-theories, with the important difference that it couples only to electromagnetic energy. Since the dilaton field couples to *all* the matter (although generally to different sectors with different powers) then the strong and electroweak charges, as well as particle masses, can also vary with x^μ. These similarities highlight the deep connections between effective fundamental theories in higher dimensions and varying-constant theories, [14].

To obtain the cosmological equations we vary the action with respect to the metric to give the generalised Einstein equations

$$G_{\mu\nu} = 8\pi G\left(T_{\mu\nu}^{mat} + T_{\mu\nu}^{\psi} + T_{\mu\nu}^{em}e^{-2\psi}\right), \tag{3.2}$$

and vary with respect to the ψ field to give the equations of motion for the field that carries the α variations:

$$\Box\psi = \frac{2}{\omega}e^{-2\psi}\mathcal{L}_{em}. \tag{3.3}$$

It is clear that $\mathcal{L}_{em}$ vanishes for a sea of pure radiation since then $\mathcal{L}_{em} = (E^2 - B^2)/2 = 0$. This suggests a negligible change in e in the radiation epoch, a fact confirmed by our numerical calculations. The only significant contribution to a variation in ψ comes from nearly pure electrostatic or magnetostatic energy.

In the matter epoch changes in e will occur. In order to make quantitative predictions we need to know how non-relativistic matter contributes to the RHS of Eqn. (3.3). This is parametrised by the ratio $\zeta = \mathcal{L}_{em}/\rho$, where ρ is the energy density, and for baryonic matter $\mathcal{L}_{em} \approx E^2/2$. For protons and neutrons ζ_p and ζ_n can be *estimated* from the electromagnetic corrections to the nucleon mass, 0.63 MeV and −0.13 MeV, respectively [15]. This correction contains the $E^2/2$ contribution (always positive), but also terms of the form $j_\mu a^\mu$ (where j_μ is the quarks' current) and so cannot be used directly. Hence we take a guiding value $\zeta_p \approx \zeta_n \sim 10^{-4}$. Furthermore the cosmological value of ζ (denoted ζ_m) has to be weighted by the fraction of matter that is non-baryonic, a point ignored in the literature [12, 18]. Hence, ζ_m depends strongly on the nature of the dark matter, and it could be that $\zeta_{CDM} \approx -1$ (e.g. superconducting cosmic strings, for which $\mathcal{L}_{em} \approx -B^2/2$), or $|\zeta_{CDM}| \ll 1$ (neutrinos). BBN predicts an approximate value for the baryon density of $\Omega_B \approx 0.03$ with a Hubble parameter of $h_0 \approx 0.6$, implying $\Omega_{CDM} \approx 0.3$. Hence, depending on the nature of the dark matter, ζ_m can be positive or negative and have a modulus between 0 and ≈ 1.

Assuming a spatially-flat, homogeneous and isotropic Friedmann metric with expansion scale factor $a(t)$ we obtain the Friedmann equation[1] $(G = c \equiv 1)$

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3}\left(\rho_m\left(1 - |\zeta_m| + |\zeta_m|e^{-2\psi}\right) + \rho_r e^{-2\psi} + \rho_\psi\right) + \frac{\Lambda}{3} \tag{3.4}$$

[1] We refer to appendix A for a more rigorous form of this equation

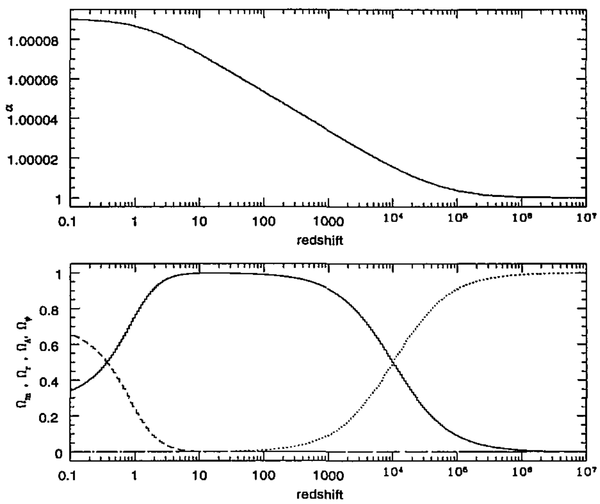

Figure 3.1: Cosmological evolution from radiation domination through matter domination and into lambda domination with coupling $\zeta_m/\omega = -0.02\%$. The upper graph plots α as a function of the redshift z. The lower graph shows the energy densities of radiation (.....), dust (——), cosmological constant (- - -) and the scalar field (combined) as fractions of the total energy density. The scalar field energy is subdominant at all times. α increases in the matter era, but approaches a constant after Λ takes over the expansion.

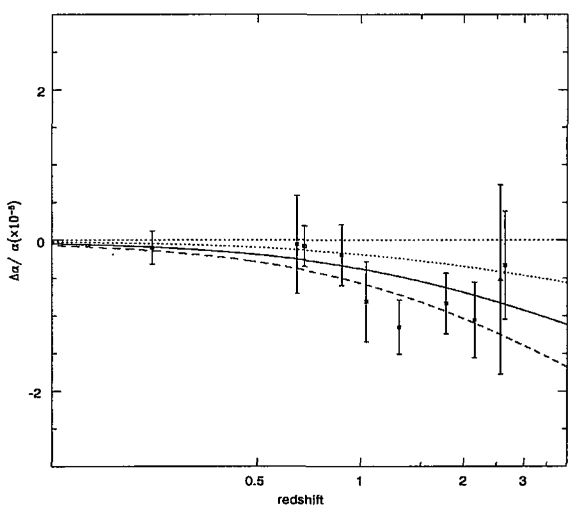

Figure 3.2: The data points are the QSO results for the changing $\alpha(z)$ reported in refs.[2, 1, 3]. The solid line is the theoretical prediction for $\alpha(z)$ in our model with $\zeta_m/\omega = -0.02\%$. The top (.....) and bottom (- - -) lines correspond to choices $\zeta_m/\omega = -0.01\%$ and $\zeta_m/\omega = -0.03\%$ respectively

where the cosmological vacuum energy ρ_Λ is a constant given by $\Lambda/(8\pi)$, and $\rho_\psi = \frac{\omega}{2}\dot{\psi}^2$. For the scalar field we get

$$\ddot{\psi} + 3H\dot{\psi} = -\frac{2}{\omega}e^{-2\psi}\zeta_m\rho_m \tag{3.5}$$

where $H \equiv \dot{a}/a$. The conservation equations for the non-interacting radiation and matter densities, ρ_r and ρ_m respectively, are:

$$\begin{aligned} \dot{\rho_m} + 3H\rho_m &= 0 && (3.6) \\ \dot{\rho}_r + 4H\rho_r &= 2\dot{\psi}\rho_r. && (3.7) \end{aligned}$$

This last relation can be written as

$$\dot{\tilde{\rho}}_r + 4H\tilde{\rho}_r = 0, \tag{3.8}$$

with $\tilde{\rho}_r \equiv \rho_r e^{-2\psi} \propto a^{-4}$. Eq. (3.5) may be expressed in terms of the kinetic energy density of the ψ field, $\rho_\psi = \omega\dot{\psi}^2/2$, to give

$$\dot{\rho_\psi} + 6H\rho_\psi = 2\sqrt{\frac{2}{\omega}}e^{-2\psi}\zeta_m\rho_m\sqrt{\rho_\psi}. \tag{3.9}$$

The ψ field behaves like a stiff Zeldovich fluid with $\rho_\psi \propto a^{-6}$ when the RHS vanishes.

Eqns. (3.4-3.7), govern the Friedmann universe with time-varying $\alpha = \exp(2\psi)e_0^2/\hbar c$. They depend on the choice of the parameter ζ_m/ω, which we take to be negative. We evolve these equations numerically from early radiation-domination, through the matter era and into vacuum domination by ρ_Λ. Fig. 3.1 shows the evolution of α with redshift in this model, for $\zeta_m/\omega = -0.02\%$. We note that ψ and α remain almost constant during early radiation domination where baryonic species become relativistic. In the matter epoch, ψ and α increase slightly towards lower redshifts, but tend to constant values again once the universe starts accelerating, and Λ dominates - this is due to the friction term $H\dot{\psi}$ in Eq. (3.5). This Λ effect reduces variations in α during the last expansion time of our universe where the local geonuclear effects of varying

α are strongly constrained by observations, [16, 17], while allowing the cosmological variations observed by [2, 1, 3] at redshifts, $z \approx 0.5 - 3.5$, where the effects of Λ on the expansion progressively diminish. In Figure 3.2 we plot the predicted change in α for $-\zeta_m/\omega = 0.01, 0.02, 0.03\%$, and the binned QSO data from refs.[2, 1, 3]. Given the uncertainties in ζ_m discussed above, it is possible to fit the data with $\omega = \mathcal{O}(1)$, so that the theory's length scale is of the order of the Planck length.

In view of the special $\alpha(z)$ variation produced by the cosmic acceleration there is agreement with all laboratory, geological and astrophysical constraints on varying-α deriving from the last expansion time (cf. [18, 19, 16, 17]). Notice also that the supernovae luminosity data are fitted by our model, since ψ affects the cosmological expansion very little, and its direct effect upon the luminosities of astrophysical objects is negligible. Hence, our Hubble diagram is precisely the same as that of a universe with constant α and $\Omega_m \approx 0.3$ and $\Omega_\Lambda \approx 0.7$. Our model also meets constraints from BBN, since it occurs deep in the radiation epoch, $z \sim 10^9 - 10^{10}$, when α is predicted to be only 0.007% lower than today. The standard BBN scenario can withstand variations in α of the order of 1% without contradicting observations (see [20] and references therein). The value of α at CMB decoupling, $z \approx 1000$ is only $\sim 0.005\%$ lower than today, compatible with CMB observations, [20, 21], which place an upper bound of a few percent. However, the variations we predict are close enough to these limits to hold out the possibility of observational test in the future by more detailed calculations of the effects on BBN and the CMB, and more precise data. Low-redshift observations of molecular and atomic transitions [22] can provide important information about the value of α close to the redshift where acceleration commences, $z_\Lambda \sim 0.7$, if the chemical isotopic evolution uncertainty can be reduced [23].

3.4 Spatial variations in α

Spatial variations of α are likely to be significant [24], and our model makes firm predictions on how α varies near massive objects. Linearising eq. (3.3) and following the calculation of [25], one finds that relative variations in α are proportional to the local gravitational potential:

$$\frac{\Delta\alpha}{\alpha} = -\frac{\zeta_s}{\pi\omega}\frac{GM}{r} \approx 10^{-4}\frac{\zeta_s}{\zeta_m}\frac{GM}{r} \tag{3.10}$$

where M is the mass of the compact object, r is its radius, and ζ_s is its value of ζ. When ζ_m and ζ_s have different signs, for a cosmologically *increasing* α, we predict that α should *decrease* on approach to a massive object. If $|\zeta_m| \approx \zeta_s$, on extragalactic scales the CMB temperature anisotropy $\Delta T/T \sim GM/r$ would lead us to expect large-scale spatial gradients of order $\Delta\alpha/\alpha \sim 10^{-9}$. More locally, one would need an object not larger than some tens of Schwarzschild radii for the effect on $\alpha(r)$ to be observable with current technology. However with improved technology, one might find less demanding candidates. An independent low-z test of the effects seen by [2, 1] could ultimately be provided by the detection of a $\Delta\alpha \neq 0$ effect from the fine structure of stellar spectral lines. The exact relation between the change in α with redshift and in space (near massive objects) is model dependent [25], but eq.(3.10) provides the exact prediction for the simple varying-α theory considered in this chapter.

3.5 Sensitivity to Fifth Force Experiments

Spatial gradients in α lead to an extra force acting upon matter coupling to ψ via the f_{em}^2 term. In order to compute this force one must model ζ for test bodies. The test-particle lagrangian may be split as $\mathcal{L}_t = \mathcal{L}_m + e^{-2\psi}\mathcal{L}_{em}$. Variation with respect to the metric leads to a similar split of the stress-energy tensor, producing an energy density of the form $\rho((1-|\zeta_t|) + |\zeta_t|e^{-2\psi})$, and so a mass of $m((1-|\zeta_t|) + |\zeta_t|e^{-2\psi})$. In order to preserve their ratios of $\zeta_t = \mathcal{L}_{em}/\rho$ test particles may thus be represented by

$$\mathcal{L}(y) = -\int d\tau\; m((1-|\zeta_t|) - |\zeta_t|e^{-2\psi})[-g_{\mu\nu}\dot{x}^\mu\dot{x}^\nu]^{\frac{1}{2}}\frac{\delta(x-y)}{\sqrt{-g}} \tag{3.11}$$

where over-dots are derivatives with respect to the proper time τ. This leads to equations of motion:

$$\ddot{x}^\mu + \Gamma^\mu_{\alpha\beta}\dot{x}^\alpha\dot{x}^\beta + \frac{2\zeta_t e^{-2\psi}}{(1-|\zeta_t|)-|\zeta_t|e^{-2\psi}}\partial^\mu\psi = 0 \tag{3.12}$$

which in the non-relativistic limit (with $|\zeta_t| \ll 1$) reduce to

$$\frac{d^2x^i}{dt^2} = -\nabla_i\phi - 2\zeta_t\nabla_i\psi\;, \tag{3.13}$$

where ϕ is the gravitational potential. Thus we predict an anomalous acceleration:

$$a = \frac{M_s}{r^2}\left(1 + \frac{\zeta_s\zeta_t}{\omega\pi}\right) \tag{3.14}$$

Since $\zeta_p \neq \zeta_n$ we see that ζ_t depends on the make up of the test particles. Thus this force acts differently on matter with different composition leading to violations of the weak equivalence principle [15, 26]. These are parameterized by the Eötvös parameter, which in our theory is

$$\eta \equiv \frac{2|a_1-a_2|}{a_1+a_2} = \frac{\zeta_E|\zeta_1-\zeta_2|}{\omega\pi} = \frac{\zeta_E|\zeta_1-\zeta_2|}{\pi\zeta_p}\frac{\zeta_p}{\zeta_m}\frac{\zeta_m}{\omega} \tag{3.15}$$

where E denotes the Earth and 1 and 2 two different test bodies. If $\zeta_n \approx \zeta_p \approx |\zeta_p - \zeta_n|$ the first factor is $\mathcal{O}(10^{-5})$ for typical substances used in experiments. The third factor, ζ_m/ω, is of the order of -10^{-4}. Hence for $|\zeta_m| = \mathcal{O}(0.1) - \mathcal{O}(1)$ we have consistency with the current experimental bound, $\eta < 10^{-13}$ [27]. We note that the next generation of Eötvös experiments should be able to detect the variations in α predicted by this theory, but firmer predictions require better theoretical calculations of ζ for neutrons, protons, nuclei and atoms (the uncertainties of which were discussed above).

3.6 Predictions for Future Varying-α Probes

Due to the potentially enormous impact of the Webb et. al. results [1, 2] it is vital that the searches be independently confirmed and extended to higher redshift. In an attempt to aid future observational efforts, we will in this section give more extensive quantitative predictions of our varying α theory. We use a wide range of cosmologies from dust-only Universes to models strongly dominated by a cosmological constant. We also explore results from using different values for the model parameter determining the amount of change in α.

Table 3.1 shows best fit[2] to the Webb data with cosmological parameters ranging from 100% dust, to 90% cosmological constant. The best fit is achieved by finding optimal value for the parameter ζ_m/ω for the given scenario. Table 3.2 shows results for models assuming the currently preferred energy distribution of 30% matter and 70% dark energy. The different columns show various choices for the parameter ζ_m/ω. Both tables show predicted variation for $z = 0.1$ (geophysical constraints), $z = 0.5$ to 3 (current quasar absorption line observations) and goes on to predict variations for higher redshift $z = 3 - 6$ possibly subject to future observations.

3.7 Conclusions

In summary, we have shown how a cosmological generalisation of Bekenstein's theory of a varying e can naturally explain the reported variations in the fine structure constant whilst satisfying all other observational bounds. The onset of Λ domination is shown to be closely related to the cosmic epoch when significant changes in α cease to occur. Our numerical results show that with a natural coupling, and using observational constraints on the nature of the cold dark matter, α changes significantly only in the matter dominated epoch. At the onset of Λ domination, the expansion accelerates and α rapidly approaches a constant. This model also places specific restrictions on the nature of the dark matter.

[2] We used a Least Sum of Squares method to fit the model to the data. The values of the Sum of Squares for various sets of cosmological parameters are shown as "SS" in table 3.1

Redshift z	$\Delta\alpha/\alpha(\times 10^{-5})$ $\Omega_m = 0.1$ $\Omega_\Lambda = 0.9$ $\zeta_m/\omega = 0.029\%$ $SS = 1.38 \cdot 10^{-10}$	$\Delta\alpha/\alpha(\times 10^{-5})$ $\Omega_m = 0.3$ $\Omega_\Lambda = 0.7$ $\zeta_m/\omega = 0.020\%$ $SS = 1.24 \cdot 10^{-10}$	$\Delta\alpha/\alpha(\times 10^{-5})$ $\Omega_m = 0.5$ $\Omega_\Lambda = 0.5$ $\zeta_m/\omega = 0.017\%$ $SS = 1.17 \cdot 10^{-10}$	$\Delta\alpha/\alpha(\times 10^{-5})$ $\Omega_m = 0.7$ $\Omega_\Lambda = 0.3$ $\zeta_m/\omega = 0.016\%$ $SS = 1.17 \cdot 10^{-10}$	$\Delta\alpha/\alpha(\times 10^{-5})$ $\Omega_m = 1.0$ $\Omega_\Lambda = 0.0$ $\zeta_m/\omega = 0.015\%$ $SS = 1.15 \cdot 10^{-10}$
0.1	-0.018	-0.032	-0.048	-0.057	-0.062
0.5	-0.133	-0.196	-0.216	-0.243	-0.258
1.0	-0.319	-0.381	-0.411	-0.422	-0.434
1.5	-0.495	-0.542	-0.562	-0.573	-0.573
2.0	-0.671	-0.691	-0.690	-0.690	-0.692
2.5	-0.837	-0.819	-0.800	-0.791	-0.785
3.0	-0.984	-0.934	-0.899	-0.881	-0.868
3.5	-1.128	-1.032	-0.996	-0.958	-0.939
4.0	-1.243	-1.110	-1.063	-1.031	-1.007
4.5	-1.370	-1.197	-1.137	-1.099	-1.070
5.0	-1.463	-1.275	-1.204	-1.161	-1.127
5.5	-1.564	-1.344	-1.263	-1.215	-1.177
6.0	-1.650	-1.403	-1.314	-1.261	-1.220
1100	-7.803	-5.542	-4.957	-4.601	-4.301
10^{10}	-10.717	-7.649	-6.682	-6.181	-5.760

Table 3.1: Predicted variation in the fine structure constant at redshifts 0 - 6, last scattering ($z = 1100$) and BBN ($z \sim 10^{10}$), for flat $k = 0$ Universes with different values of Ω_m and Ω_Λ. The results represents optimal values for ζ/ω for the different sets of cosmological parameters. The goodness-of-fit is indicated by the Sum of Squares, SS

Redshift z	$\Delta\alpha/\alpha(\times 10^{-5})$ $\zeta/\omega = 0.01\%$	$\Delta\alpha/\alpha(\times 10^{-5})$ $\zeta/\omega = 0.02\%$ (best fit)	$\Delta\alpha/\alpha(\times 10^{-5})$ $\zeta/\omega = 0.03\%$	$\Delta\alpha/\alpha(\times 10^{-5})$ $\zeta/\omega = 0.04\%$
0.1	-0.015	-0.032	-0.047	-0.063
0.5	-0.098	-0.196	-0.294	-0.393
1.0	-0.190	-0.381	-0.571	-0.761
1.5	-0.270	-0.542	-0.812	-1.083
2.0	-0.345	-0.691	-1.036	-1.381
2.5	-0.409	-0.819	-1.227	-1.636
3.0	-0.466	-0.934	-1.399	-1.866
3.5	-0.515	-1.032	-1.546	-2.061
4.0	-0.554	-1.110	-1.663	-2.217
4.5	-0.597	-1.197	-1.792	-2.390
5.0	-0.636	-1.275	-1.910	-2.546
5.5	-0.671	-1.344	-2.013	-2.684
6.0	-0.700	-1.403	-2.101	-2.801
1100	-2.825	-5.642	-8.462	-11.283
10^{10}	-3.824	-7.649	-11.473	-15.296

Table 3.2: Predicted variation in the fine structure constant at redshifts 0 - 6, last scattering ($z = 1100$) and BBN ($z \sim 10^{10}$), for flat $k = 0$ Universes with $\Omega_m = 0.3$ and $\Omega_\Lambda = 0.7$. The different scenarios represent different choices for the value of our model parameter ζ/ω

Our model complies with geonuclear constraints, like Oklo, but is consistent with the non-zero variations in $\alpha(z)$ inferred from observations of quasar absorption lines [2, 1, 3] at $z \approx 0.5 - 3.5$. It is also consistent with CMB and BBN observational constraints. The model is attractive because of its simplicity; apart from the (observed) cosmological constant value, the only free parameter introduced is an energy scale similar to the Planck scale. There is also only one extra scalar field, and no potential has to be put in by hand. Further tests for this model will be possible using stellar spectra and the next generation of Eötvös experiments.

In the hope of aiding future observational efforts we have also used our model to make predictions for variations at higher redshift for a range of cosmological and model parameters.

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Chapter 4

Further Properties of Varying-Alpha Cosmologies

4.1 Introduction

One of the problems that cosmologists have faced in their attempts to assess the astronomical consequences of a time variation in the fine structure constant, α, has been the absence of an exact theory describing cosmological models in the presence of varying α. This is one major motivation for the work presented in this thesis. In chapter 3 we extended the generalisation of Maxwell's equations developed by Bekenstein so that one can explore the behaviour of varying-α cosmologies self-consistently. We reported numerical studies of the cosmological evolution of varying-α cosmologies with zero curvature, non-zero cosmological constant, and matter density matching observations. They reveal important properties of varying-α cosmologies that are shared by other theories in which 'constants' vary via the propagation of a causal scalar field obeying 2nd-order differential equations.

In this chapter we present a detailed analytic and numerical study of the behaviour of the cosmological solutions of the varying-α theory presented in the previous chapter. We shall confine our attention to universes containing dust and radiation but analyse the effects of negative spatial curvature and a positive cosmological constant. Extensions to general perfect-fluid cosmologies can easily be made if required.

We should not confuse this theory with other similar variations. Bekenstein's theory [1] does not take into account the stress energy tensor of the dielectric field in

Einstein's equations, and their application to cosmology. A supersymmetric extension to Bekenstein's and our framework has been provided by Olive et. al. [2]. Dilaton theories predict a global coupling between the scalar and all other matter fields. As a result they predict variations in other constants of nature, and also a different dynamics to all the matter coupled to electromagnetism. An interesting application of our approach has also recently been made to braneworld cosmology in [3]. More phenomenological models have been suggested [4, 5] which expand α around its present value,

$$\alpha = \alpha(0) + \lambda \frac{\phi}{M_{Pl}}, \tag{4.1}$$

and examine the inferred violations of the weak equivalence principle. These models also suggest identifying the scalar field responsible for the α-variations with the quintessence field causing the current acceleration of the Universe, thereby constraining the class of viable quintessence potentials. Chiba[5] also noted that combining the Quasar results with the Oklo data, required a slowing down of the scalar field variation. As seen in Chapter 3 this is naturally explained in our model by the onset of Λ-dominated expansion.

4.2 The Cosmological Equations

The Einstein equations in our generalised Bekenstein theory were developed in the previous chapter, as well as the resulting Friedmann equations which we recapitulate here: Assuming a homogeneous and isotropic Friedmann metric with expansion scale factor $a(t)$ and curvature parameter k in eqn. (3.2), we obtain the field equations ($c \equiv 1$)

$$\begin{aligned} \left(\frac{\dot{a}}{a}\right)^2 &= \frac{8\pi G}{3}\left(\rho_m\left(1 - |\zeta_m| + |\zeta_m| \exp\left[-2\psi\right]\right) + \rho_r \exp\left[-2\psi\right] + \frac{\omega}{2}\dot{\psi}^2\right) \\ &\quad - \frac{k}{a^2} + \frac{\Lambda}{3}, \end{aligned} \tag{4.2}$$

where Λ is the cosmological constant. For the scalar field we have the propagation equation,

$$\ddot{\psi} + 3H\dot{\psi} = -\frac{2}{\omega}\exp\left[-2\psi\right]\zeta_m\rho_m, \tag{4.3}$$

where $H \equiv \dot{a}/a$ is the Hubble expansion rate. Note that the sign of the evolution of ψ is dependent on the sign of ζ_m. Since the observational data is consistent with a **smaller** value of α in the past, we will in this chapter confine our study to **negative** values of ζ_m, in line with our discussion in chapter 3. The conservation equations for the non-interacting radiation and matter densities are

$$\begin{aligned} \dot{\rho_m} + 3H\rho_m &= 0 & (4.4) \\ \dot{\rho_r} + 4H\rho_r &= 2\dot{\psi}\rho_r. & (4.5) \end{aligned}$$

and so $\rho_m \propto a^{-3}$and $\rho_r\ e^{-2\psi} \propto a^{-4}$, respectively. If additional non-interacting perfect fluids satisfying equation of state $p = (\gamma - 1)\rho$ are added to the universe then they contribute density terms $\rho \propto a^{-3\gamma}$ to the RHS of eq.(4.2) as usual. This theory enables the cosmological consequences of varying e, to be analysed self-consistently rather than by changing the constant value of e in the standard theory to another constant value, as in the original proposals made in response to the large numbers coincidences (see ref. [6] for a full discussion).

We have been unable to solve these equations in general except for a few special cases. However, as with the Friedmann equation of general relativity, it is possible to determine the overall pattern of cosmological evolution in the presence of matter, radiation, curvature, and positive cosmological constant by matched approximations. We shall consider the form of the solutions to these equations when the universe is successively dominated by the kinetic energy of the scalar field ψ, pressure-free matter, radiation, negative spatial curvature, and positive cosmological constant. Our analytic expressions are checked by numerical solutions of (4.2) and (4.3), where we have employed the NAG routines D02PCF and D02PVF for the integration tasks.

4.2.1 The Dust-dominated era

We consider first the behaviour of dust-filled universes far from the initial singularity. We assume that $k = 0 = \Lambda = \rho_\gamma$, so the Friedmann equation (4.2) reduces to

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\left(\rho_m\left(1 - |\zeta_m| + |\zeta_m| \exp\left[-2\psi\right]\right) + \frac{\omega}{2}\dot{\psi}^2\right), \tag{4.6}$$

and seek a self-consistent approximate solution in which the scale factor behaves as

$$a = t^{2/3} \tag{4.7}$$

$$\frac{d}{dt}(\dot{\psi}a^3) = N \exp\left[-2\psi\right] \tag{4.8}$$

where

$$N \equiv -\frac{2\zeta_m}{\omega}\rho_m a^3 \tag{4.9}$$

is a positive constant since we have confined ourselves to $\zeta_m < 0$. If we put

$$x = \ln(t)$$

then (4.8) becomes

$$\psi'' + \psi' = N \exp[-2\psi] \tag{4.10}$$

with $N \geq 0$ and $' \equiv d/dx$. This equation has awkward behaviour. For any power-law behaviour of the scale factor other than (4.7) a simple exact solution of (4.8) exists. However, the late-time dust solutions are exceptional, reflecting the coupling of the charged matter to the variations in ψ, and are approximated by the following asymptotic series:

$$\psi = \frac{1}{2}\ln[2Nx] + \sum_{n=1}^{\infty} a_n x^{-n} \tag{4.11}$$

To see this, substitute this in the evolution eqn. (4.10) for ψ then it becomes:

$$\begin{aligned} -\frac{1}{2x^2} + \sum_{n=1} n(n+1)a_n x^{-n-2} & \\ +\frac{1}{2x} - \sum_{n=1} n a_n x^{-n-1} &= \frac{1}{2x}\exp[-2\sum_{n=1} a_n x^{-n}] \end{aligned} \tag{4.12}$$

Now we can pick the a_n to cancel out all the terms in x^{-r} , $r \geq 2$ on the left-hand side. This requires

$$a_2 = a_1 = -\frac{1}{2}, a_3 = 2a_2, a_4 = 3a_3 = 3 \times 2a_2, etc$$

hence

$$\begin{aligned} \sum_{n=1} a_n x^{-n} &= -\frac{1}{2}\{\frac{1}{x} + \frac{1}{x^2} + \frac{2}{x^3} \\ &+ \frac{2\times 3}{x^4} + \frac{2\times 3\times 4}{x^5} + ... + \frac{(r-1)!}{x^r} + ..\} \end{aligned}$$

all that is left of the eqn. (4.12) is

$$\frac{1}{2x} = \frac{1}{2x}\exp[-2\sum_{n=1} a_n x^{-n}] \rightarrow \frac{1}{2x}$$

as $x \to \infty$. So, at late times, as $x = \ln(t)$ becomes large, we have

$$\begin{aligned} \psi &= \frac{1}{2}\ln[2N(\ln(t))] - \frac{1}{2}\{\frac{1}{\ln(t)} + \frac{1}{(\ln(t))^2} + \frac{2}{(\ln(t))^3} \\ &+ \frac{2\times 3}{(\ln(t))^4} + \frac{2\times 3\times 4}{(\ln(t))^5} + ... + \frac{(r-1)!}{(\ln(t))^r} + ..\}; \end{aligned} \tag{4.13}$$

also, since $\alpha = \exp[2\psi]$ we have, as $t \to \infty$

$$\begin{aligned} \alpha &= 2N\ln(t) \times \exp[-\frac{1}{\ln(t)} - \frac{1}{(\ln(t))^2} - \frac{2}{(\ln(t))^3} \\ &- \frac{2\times 3}{(\ln(t))^4} - \frac{2\times 3\times 4}{(\ln(t))^5} - ... - \frac{(r-1)!}{(\ln(t))^r} -]. \end{aligned} \tag{4.14}$$

So, to leading order, we have

$$\alpha \sim 2N \ln(t) \exp[-\frac{1}{\ln(t)}] \tag{4.15}$$

The non-analytic $\exp[1/x]$ behaviour shows why the eqn. (4.10), despite looking simple, has awkward behaviour. We can simplify the asymptotic series (4.14) a bit further because we know from the definition of the logarithmic integral function $li(x) = \int_0^x dt/\ln(t) = \mathrm{Ei}[\ln(x)]$, that as $x \to \infty$

$$li(x) \sim \exp[x] \sum_{n=0}^{\infty} \frac{n!}{x^{n+1}} \tag{4.16}$$

so the series we have in (4.13) in $\{..\}$ brackets is

$$\sum_{r=1}^{\infty} \frac{(r-1)!}{x^r} \sim \exp[-x] li(\exp[x]) \tag{4.17}$$

and so asymptotically

$$\psi = \frac{1}{2} \ln[2Nx] - \frac{1}{2} \exp[-x] li(\exp[x]). \tag{4.18}$$

Hence, as $t \to \infty$,

$$\psi = \frac{1}{2} \ln[2N \ln(t)] - \frac{1}{2t}\, li(t) = \frac{1}{2} \ln[2N \ln(t)] - \frac{1}{2t} \mathrm{Ei}[\ln(t)] \tag{4.19}$$

and so asymptotically,

$$\alpha = \exp[2\psi] = 2N \exp[-t^{-1} li(t)] \ln t. \tag{4.20}$$

This asymptotic behaviour is confirmed by solving equations (4.2-4.5) numerically for $\rho_m \gg \rho_r, \rho_\psi$. By using a range of initial values for ψ we produce the plot in fig (4.1), in which the asymptotic solution is clearly approached.

We need to check that the original assumption of $a = t^{2/3}$ in the Friedmann eqn. (4.2) is self consistent. The relevant terms are

$$\rho_m \left(1 - |\zeta_m| + |\zeta_m| \exp\left[-2\psi\right]\right) + \frac{\omega}{2}\dot{\psi}^2 \tag{4.21}$$

The $\exp[-2\psi] = \alpha^{-1}$ falls off as $t \to \infty$ so the $\rho_m \left(1 + \zeta_m \exp\left[-2\psi\right]\right) \propto a^{-3}$ term dominates as expected. For the kinetic term $\dot{\psi}^2$ we have

$$\dot{\psi} = \frac{1}{t} \times O(\frac{1}{\ln(t)}) \tag{4.22}$$

and so again the $\dot{\psi}^2$ term falls off faster than t^{-2} as $t \to \infty$ and the $a = t^{2/3}$ behaviour is an ever-improving approximation at late times. If we examine the form of the solution (4.20) we see that α always *increases* with time as a logarithmic power until it grows sufficiently for the exponential term on the right-hand side of (4.3) to affect the solution significantly and slow the rate of increase by the series terms. The rate at which α grows is controlled by the total density of matter in the model, which is directly proportional to the constant N, defined by eqn. (4.9). The higher the density of matter (and hence N) the faster the growth in α. However, because of the logarithmic time-variation, the dependence on ρ_m, ω, and ζ_m is weak. The self-consistency of the usual $a = t^{2/3}$ dust evolution for the scale factor leaves the standard cosmological tests unaffected. This is just as one expects for the very variations indicated by the observations of [9].

4.2.2 The Radiation-dominated era

In the radiation era we assume $k = \Lambda = 0$ and take $a = t^{1/2}$ as the leading order solution to (4.2). We must now solve

$$\frac{d}{dt}\left(\dot{\psi}a^3\right) = N \exp[-2\psi]. \tag{4.23}$$

There is a simple particular exact solution

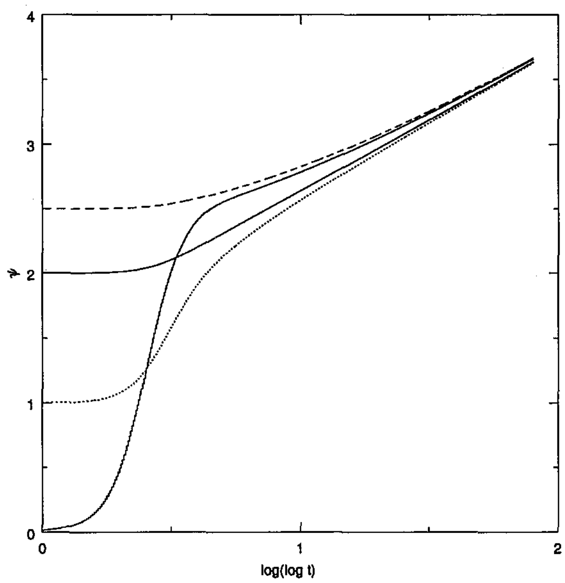

Figure 4.1: Numerical solution to the equations in the dust-dominated epoch. ψ is plotted against $log(logt)$, with initial conditions $\psi = 0, 1, 2, 2.5$. The numerical solution clearly approaches the asymptotic solution in the expected manner. The time is plotted in Planck units of $10^{-43}s$.

$$\psi = \frac{1}{2}\ln(8N) + \frac{1}{4}\ln(t) \tag{4.24}$$

Consider a perturbation of this solution by $f(t)$

$$\psi \;=\; \frac{1}{2}\ln(8N) + \frac{1}{4}\ln(t) + f(t)$$

Inserted in eqn. (4.23) we then get

$$\ddot{f} + \frac{3}{2t}\dot{f} = \frac{1}{8t^2}(\exp[-2f] - 1) \tag{4.25}$$

Let us first consider the case of a large perturbation, $\exp(-2f) \ll 1$. The RHS of (4.25) then reduces to $-1/(8t^2)$, and through a straightforward integration we get

$$\dot{f} = -\frac{1}{4t} + Ct^{-3/2} \tag{4.26}$$

with C an arbitrary constant. As t increases this will approach $-1/(4t)$ which has the same absolute value and is opposite in sign to the derivative of the exact solution (4.24). Thus for values of ψ much higher than this solution $\dot{\psi}$ is zero. ψ will stay constant until the perturbation f becomes small and ψ approaches the exact solution (4.24).

To establish the stability of the exact solution we need to consider small perturbations around it. For small f we have

$$\ddot{f} + \frac{3}{2t}\dot{f} + \frac{1}{4t^2}f = 0. \tag{4.27}$$

Hence,

$$f = \frac{1}{t}\{A\sin[\sqrt{3}\ln(t)] + B\cos[\sqrt{3}\ln(t)] \tag{4.28}$$

Thus, we have

$$\begin{aligned}\psi &\to \frac{1}{2}\ln(8N)+\frac{1}{4}\ln(t)\\&+\frac{1}{t}\{A\sin[\sqrt{3}\ln(t)]+B\cos[\sqrt{3}\ln(t)]\}\end{aligned}\tag{4.29}$$

$$\begin{aligned}\alpha &= e^{2\psi}\to 8Nt^{1/2}\exp[\frac{2}{t}\{A\sin[\sqrt{3}\ln(t)]\\&+B\cos[\sqrt{3}\ln(t)]\}]\to 8Nt^{1/2}\end{aligned}\tag{4.30}$$

as $t\to\infty$.

We need to check that the $\dot{\psi}^2$ term does not dominate as $t\to\infty$. We have

$$\dot{\psi}\sim\frac{1}{4t}+\frac{1}{t^2}\times oscillations \tag{4.31}$$

Thus the $\dot{\psi}^2$ term is the **same order** of t as the radiation density term if we assume $a\sim t^{1/2}$. Also, the matter density term $\rho_m(1-|\zeta_m|+|\zeta_m|\exp[-2\psi])\sim\rho_m\exp[-2\psi]\sim a^{-3}\exp[-2\psi]\sim t^{-3/2}\times t^{-1/2}\sim t^{-2}$ is the same order of time variation as the radiation-density term because of the variation in α. The assumption $a=t^{1/2}$ is still good asymptotically but there is an algebraic constraint from the Friedmann eqn. (4.2)

Evaluating the terms in (4.2), we have

$$\frac{1}{4t^2}=\frac{8\pi G}{3}\left(\frac{M}{t^{3/2}}[1+\frac{S}{8Nt^{1/2}}]\right)+\frac{\Gamma}{t^2}+\frac{\omega}{32t^2}\tag{4.32}$$

where $\rho_m=Ma^{-3}$, $\rho_\gamma\exp[-2\psi]=\Gamma a^{-4}$, $N=-2M\zeta_m/\omega$ where from the discussion in chapter 3 we have $\zeta_m/\omega\sim-0.02\%$ with $\omega\sim 1$. So, to $O(t^{-2})$, we have the algebraic constraint

$$\frac{1}{4}=\frac{8\pi G}{3}[\frac{3\omega}{32}+\Gamma]$$

This generalises the familiar general relativity ($\omega=0$) radiation universe case where we have $\Gamma=3/32\pi G$.

Again, the asymptotic behaviour in eqns. (4.29)-(4.30), and the approach to the exact solution (4.24), can be confirmed by numerical solutions to eqns.(4.2) - (4.5) in the case of radiation domination. The results from runs with initial values for $\psi = -8, 0, 8$, $\dot{\psi} = 0$ and same value for N, are shown in fig.(4.2). The particular solution (4.24) is clearly an attractor. It is also seen that if the system starts off with values higher than $1/2 \ln(8N)$, ψ will stay constant until it reaches the value of the solution, as predicted above. In cosmological models containing matter and radiation with densities given by those observed in our universe this is the case, as seen in the computations shown in chapter 3. Hence, during the radiation era α remains approximately constant until the dust era begins.

This analysis can easily be extended to other equations of state. If the Friedmann equation contains a perfect fluid with equation of state $p = (\gamma - 1)\rho$ with $\gamma \neq 0, 1, 2$ then there is a late time solution of (4.2) and (4.3) of the form

$$a = t^{\frac{2}{3\gamma}} \tag{4.33}$$

$$\psi = \frac{1}{2} \ln[\frac{N\gamma^2}{(\gamma - 1)(2 - \gamma)}] + \left(\frac{\gamma - 1}{\gamma}\right) \ln(t) \tag{4.34}$$

which reduces to (4.24) when $\gamma = 4/3$. This solution only exists for fluids with $1 < \gamma < 2$.

4.2.3 The Curvature-dominated era

In our earlier study in chapter 3 we showed that the evolution of α stops when the universe becomes dominated by the cosmological constant. This behaviour also occurs when an open universe becomes dominated by negative spatial curvature. In a curvature-dominated era we assume that (4.2) has the Milne universe solution with

$$a = t. \tag{4.35}$$

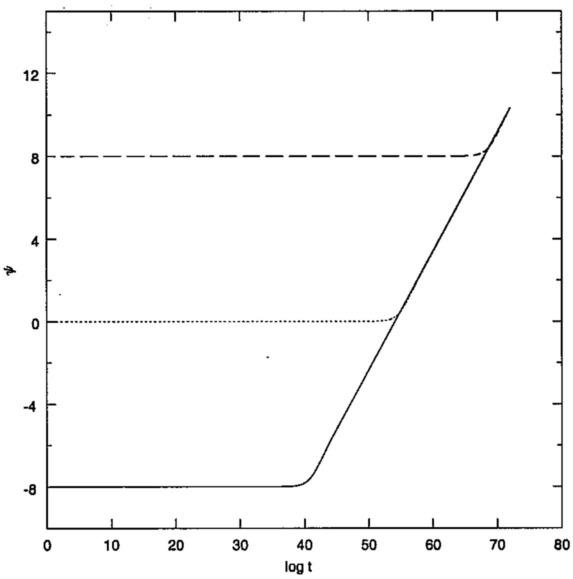

Figure 4.2: Numerical solution to the equations in the radiation-dominated epoch given different initial conditions. The particular exact solution is eventually reached in all cases. The time is plotted in units of the Planck time.

We must now solve eq. (4.23) again. It has the form

$$\frac{d}{dt}\left(\dot{\psi}t^3\right) = N\exp[-2\psi]. \tag{4.36}$$

We seek a solution of the form

$$\psi = \frac{1}{2} + f(t) \tag{4.37}$$

Hence, for small f

$$\ddot{f} + \frac{3}{t}\dot{f} + \frac{2N}{t^2}f = 0 \tag{4.38}$$

Solutions exist with $f \propto t^n$ and

$$n = -1 \pm \sqrt{1-2N} \tag{4.39}$$

Since $N > 0$ we see that the real part of n is always decaying and so

$$\psi \to const \tag{4.40}$$

as $t \to \infty$. Thus, as $t \to \infty$ we have

$$\alpha \sim \ \alpha_\infty \exp[2At^{-1\pm\sqrt{1-2N}}], \tag{4.41}$$

where α_∞ and A are constants.

Again we need to check that the $\dot{\psi}^2$ term does not come to dominate. We have $\dot{\psi}^2 \sim t^{2(n-1)}$ as $t \to \infty$ and this always falls faster than $ka^{-2} \propto t^{-2}$ since $n \leq 0$, so our approximation is always good. Thus we have shown that in open Friedmann universes α rapidly approaches a constant value after the universe becomes curvature dominated. The rate of approach is controlled by the matter density through the constant N in eq. (4.41).

This behaviour is again confirmed by numerical solution. Fig.(4.3) shows how alpha changes through the dust-epoch and how the change comes to an end as curvature takes over the expansion.

4.2.4 The Lambda-dominated era

We can prove what was displayed in the numerical results of chapter 3, and again in fig.(4.4) for the Λ-dominated era when the value of Λ matches that inferred from recent high redshift supernova observations [10]. At late times we assume the scale factor to take the form

$$a = \exp[\lambda t] \tag{4.42}$$

where $\lambda \equiv \sqrt{\frac{\Lambda}{3}}$ and so eqn.(4.2) becomes

$$\frac{d}{dt}\left(\dot{\psi} e^{3\lambda t}\right) = N \exp[-2\psi] \tag{4.43}$$

Linearising in ψ, we have

$$\ddot{\psi} + 3\lambda\dot{\psi} = N \exp[-3\lambda t]. \tag{4.44}$$

Hence,

$$\psi = \psi_0 + A \exp[-3\lambda t] - \frac{Nt}{3\lambda} \exp[-3\lambda t] \rightarrow \psi_0 \tag{4.45}$$

as $t \rightarrow \infty$, where A, ψ_0 are arbitrary constants. Thus α approaches a constant with double-exponential rapidity during a Λ-dominated phase of the universe. The dominant term controlling the late-time approach to the constant solution is proportional to the matter density via the constant N.

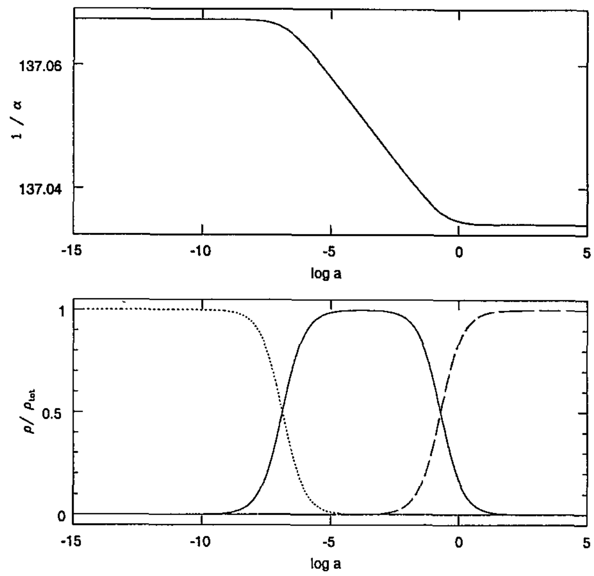

Figure 4.3: The top plot shows evolution of α from radiation domination through matter domination and into curvature domination where the change in α comes to an end. The lower plot shows radiation (dotted), matter (solid) and curvature (dashed) densities as fractions of the total energy density

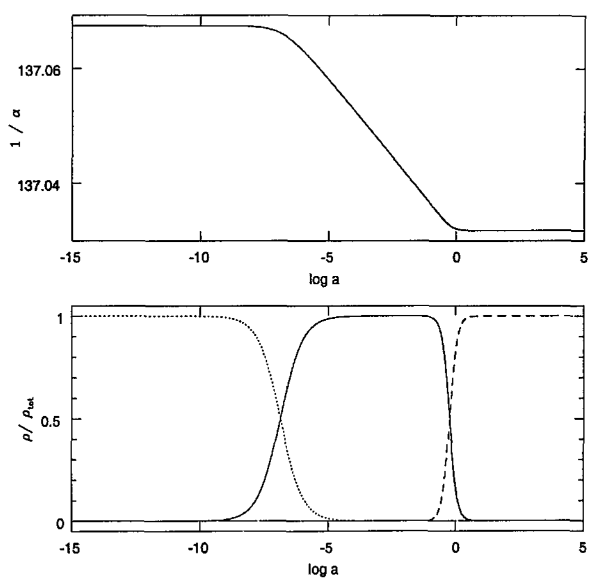

Figure 4.4: The top figure shows numerical evolution of α from radiation domination through matter domination and into lambda domination where the change in α comes to an end. The lower plot shows radiation (dotted), matter (solid) and lambda (dashed) densities as fractions of the total energy density

4.2.5 Inflationary Universes

The behaviour found for lambda-dominated universes enables us to understand what would transpire during a period of de Sitter inflation during the early stages of a varying-α cosmology. It is straightforward to extend these conclusions to any cosmology undergoing power-law inflation. Suppose the varying-α Friedmann model contains a perfect fluid with $p = (\gamma - 1)\rho$ and $0 < \gamma < 2/3$. The expansion scale factor will increase with $a(t) \propto t^{2/3\gamma}$, while ψ will be governed, to leading order by

$$(\dot{\psi}t^{2/\gamma}\dot{)} = 0 \tag{4.46}$$

Hence, for large expansion

$$\psi = \psi_0 + Dt^{-(2-\gamma)/\gamma} \to \psi_0 \tag{4.47}$$

and so ψ and α approach a constant with power-law (exponential) rapidity during any period of power-law (de Sitter) inflation. If we evaluate the kinetic term $O(\dot{\psi}^2)$ in the Friedmann equation and the terms $O(N \exp[-2\psi])$ in the ψ conservation equation, we see that the assumption of $a(t) \propto t^{2/3\gamma}$ is an increasingly good approximation as inflation proceeds. Similar behaviour would be displayed by a quintessence field which violated the strong-energy condition and came to dominate the expansion of the universe at late times. It would turn off the time variation of the fine structure constant in the same manner as the curvature of lambda terms discussed above. Note that the ψ field itself is not a possible source of inflationary behaviour in these models. We are assuming that the inflation is contributed, as usual, by some other scalar matter field with a self-interaction potential. However, if this field was charged then these conclusions could be altered as the coupling of the inflationary scalar field to the ψ field would be more complicated.

4.2.6 The Very Early Universe ($t \to 0$)

As $t \to 0$ we expect (just as in Brans-Dicke theory) to encounter a situation where the kinetic energy of ψ dominates the evolution of $a(t)$. This is equivalent to the solution approaching a vacuum solution of (4.2)-(4.3) with $\rho_m = \rho_r = 0$, as $t \to 0$. In the flat case with $\Lambda = 0$ (the $k \neq 0$ and $\Lambda \neq 0$ cases can be solved straightforwardly and the models with $\rho_r \neq 0$ can also be solved exactly in parametric form.) we have

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{4\pi G\omega}{3}\dot{\psi}^2 \tag{4.48}$$

$$\ddot{\psi} + 3H\dot{\psi} = 0 \tag{4.49}$$

Thus the exact vacuum solution is

$$\psi = \psi_0 + \frac{1}{\sqrt{12\pi G\omega}}\ln(t) \tag{4.50}$$

$$a = t^{1/3} \tag{4.51}$$

During this phase the fine structure constant *increases* as a power-law of the comoving proper time as t increases:

$$\alpha = \exp[2\psi] \propto t^{\frac{1}{\sqrt{3\pi G\omega}}} \tag{4.52}$$

Note that the matter and radiation density terms fall off slower than $\dot{\psi}^2 \propto t^{-2}$ as $t \to 0$ and $\exp[-2\psi] \propto t^{-1/(\sqrt{3\pi G\omega})}$. They will eventually dominate the evolution at some later time and the vacuum approximation will break down. As in Brans-Dicke cosmology [12] we expect the general solutions of the cosmological equations to approach this vacuum solution as $t \to 0$ and to approach the other late-time asymptotes discussed above as $t \to \infty$.

4.3 Discussion

The overall pattern of cosmological evolution is clear from the results of the last section even though it is not possible to solve the Friedmann equation exactly in most cases. There are five distinct phases:

- **a**. Near the initial singularity the kinetic part of scalar field ψ will dominate the expansion and the universe behaves like a general relativistic Friedmann universe containing a massless scalar or stiff perfect fluid field, with $a = t^{1/3}$. During this 'vacuum phase', the fine structure constant increases as a power law in time.

  b. As the universe ages the radiation density will eventually become larger than the kinetic energy of the ψ field. In this radiation dominated epoch, the fine structure constant will approach a specific solution, $\alpha \propto t^{1/2}$ asymptotically. In reality however, if the initial value of α is much larger than the specific solution, we will have a potentially very long transient period of constant evolution, and the universe may become dust dominated while α is still constant.

  c. After dust domination begins, α slowly approaches an asymptotic solution, $\alpha = 2N\ln(t) \times \exp[-t^{-1}li(t)]$, where $li(t)$ is the logarithmic integral function. If the universe has zero curvature and no cosmological constant this will approach the late time solution $\alpha \propto \ln(t)$.

  d. If the universe is open then this increase will be brought to an end when the universe becomes dominated by spatial curvature and α will approach a constant. If the curvature is positive the universe will eventually reach an expansion maximum and contract so long as there are no fluids present which violate the strong energy condition. The behaviour of closed universes also offers a good approximation to the evolution of bound spherically symmetric density inhomogeneities of large scale in a background universes and will be discussed in a later paper.

e. If there is a positive cosmological constant, the change in α will be halted when the cosmological constant starts to accelerate the universe. If any other quintessential perfect fluid with equation of state satisfying $p < -\rho/3$ is present in the universe then it would also ultimately halt the change in α when it began to dominate the expansion of the universe.

To obtain a more holistic picture of the evolution it is useful to string these different parts together. To a good approximation we know that in the vacuum phase from the Planck time t_p until t_v we have

$$a \propto t^{\frac{1}{3}}; \alpha \propto t^A; A = \frac{1}{\sqrt{3\pi G\omega}} \tag{4.53}$$

In the radiation era we have α constant until the growth kicks in at a time t_{growth}. The fine structure constant then increases as

$$a \propto \alpha \propto t^{1/2} \tag{4.54}$$

until $t_{eq}$when the radiation era end and dust takes over. However, in universes like our own, this growth era is never reached. Then, in the dust era,

$$\alpha \propto \ln t \tag{4.55}$$

until the curvature or lambda eras begin at t_c or t_Λ, after which α remains constant until the present , t_0. So, matching these phases of evolution together we can express $\alpha(t_0)$ in terms of $\alpha(t_p)$:

When the universe is open with $\Lambda = 0$:

$$\alpha(t_0) = \alpha(t_p) \left(\frac{t_v}{t_p}\right)^A \left(\frac{t_{eq}}{t_{growth}}\right)^{1/2} \left(\frac{\ln(t_c/t_p)}{\ln(t_{eq}/t_p)}\right), \tag{4.56}$$

where we have used the fact that our log formula to express ages in Planck time units.

When the universe is flat with $\Lambda > 0$:

$$\alpha(t_0) = \alpha(t_p)\left(\frac{t_p}{t_v}\right)^A \left(\frac{t_{eq}}{t_{growth}}\right)^{1/2} \left(\frac{\ln(t_\Lambda/t_p)}{\ln(t_{eq}/t_p)}\right) \tag{4.57}$$

and t_c has been replaced by t_Λ.

For the radiation era we consider two extreme cases. We look at a constant α scenario with $t_{growth} = t_{eq}$ and a scenario where it grows throughout the radiation era, $t_{growth} = t_v$.

Typically, $t_c/t_p \sim t_\Lambda/t_p \sim 10^{59}$ and $t_{eq}/t_p \sim 10^{53}$, so in both cases for constant α evolution in the radiation epoch we get

$$\alpha(t_0) = \alpha(t_p)\left(\frac{t_v}{t_p}\right)^A \left(\frac{59}{53}\right) \sim 1.11\alpha(t_p)\left(\frac{t_v}{t_p}\right)^A \tag{4.58}$$

We approximate the value for $t_v \sim t_p \sim 1$, so for continuous growth through radiation epoch we get

$$\alpha(t_0) = \alpha(t_p)\left(\frac{t_v}{t_p}\right)^A \left(10^{53}\right)^{1/2} \left(\frac{59}{53}\right) \sim 10^{26}\alpha(t_p)\left(\frac{t_v}{t_p}\right)^A \tag{4.59}$$

Hence there are very different possibilities for the change in α depending on the evolution in the radiation era.

We have proved this sequence of phases by an exhaustive numerical and analytical study. The ensuing scenario finds two interesting applications, with which we conclude.

In chapter 3 we found that our theory could fit simultaneously the varying α results reported in [8, 7, 9] and the evidence for an accelerating universe presented in [10]. We noted the curious fact that there is a coincidence between the redshift at which the universe starts accelerating and the redshift where variations in α have been observed but below which α must stabilise to be in accord with geochemical evidence [13, 14]. This may be explained dynamically in our theory by the fact that the onset of lambda domination suppresses variations in α. Therefore α remains almost constant in the radiation era, undergoes small logarithmic time-increase in the matter era, but approaches a constant value when the universe starts accelerating because of the presence of a positive cosmological constant. Hence, we comply with geological, nucleosynthesis, and microwave background radiation constraints on time-variations in α, while fitting simultaneously the observed accelerating universe and the recent high-redshift evidence for small α variations in quasar spectra.

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Chapter 5

Variations of Alpha in Space and Time

5.1 Introduction

In chapters 3 and 4 we have discussed the behaviour of a class of cosmologies in an exact theory in which the fine structure "constant" varies in time. This theory of Sandvik, Barrow and Magueijo is an extension, to include the self-gravitation of the dielectric medium, of Bekenstein's prescription [1] for generalising Maxwell's equations to incorporate varying electron charge. Henceforth we will refer to it as the BSBM theory. The fine structure "constant" α varies through the space-time dynamics of a scalar "'dielectric" field ψ, (where $\alpha = \exp[2\psi]$) in these theories. However the overall behaviour is significantly affected by the form of the coupling. Even though the requirement that the energy in ψ be positive definite fixes the sign of the coupling constant ω we find that ψ is driven by a term of the form $\mathcal{L}_{em}/\omega$, where $\mathcal{L}_{em}$ is the electromagnetic Lagrangian. In general, $\mathcal{L}_{em}$ can be positive or negative, a fact we parameterize in terms of $\zeta = \mathcal{L}_{em}/\rho$, where ρ is the energy density. The sign of ζ for the dark matter in the universe turns out to be of exceptional significance. In accordance with the work presented in earlier chapters we will also in this chapter confine ourselves to the case where $\zeta < 0$.

The studies described in previous chapters have been performed in the context of an exact isotropic and homogeneous Friedmann universe. All variations in the fine structure "constant" therefore depend only on cosmic time. However, the rate of variation that is suggested by recent astronomical observations of quasar spectra, or allowed by geophysical data at recent times, is very small, $\Delta\alpha/\alpha \sim 10^{-5}$, and spatial variations in the rate of time variation could easily be of similar order [2]. It is therefore important to determine if spatial variations in the rate of change of α are significant in the BSBM theory and whether they allow different modes of time variation to occur in addition to those studied in the purely homogeneous variations found. Such is the purpose of this chapter.

5.2 Inhomogeneous solutions with varying α

The Friedmann models with varying α have shown that when $\zeta < 0$ the homogeneous motion of the ψ does not in general create significant metric perturbations at late times and we can safely assume that the expansion scale factor is that of the usual Friedmann universe for the appropriate fluid. The behaviour of ψ then follows from a solution of the ψ conservation equation in which the expansion scale factor is taken to be that of the Friedmann universe for matter with the same equation of state in general relativity ($\psi = \zeta = 0$). Our analyses in previous chapters found that ψ is approximately constant during the radiation era, and α increases as $2N\ln(t)$ during the dust dominated era when spatial curvature is negligible, and tends to a constant in any subsequent era dominated by negative spatial curvature or a positive cosmological constant. When $\zeta < 0$ we can use the same test-motion approach to investigate inhomogeneous variations in ψ and α as the universe expands. We assume that the expansion scale factor is that of the Friedmann model

$$a = t^n \tag{5.1}$$

and solve the wave equation in one of its appropriate forms:

$$\Box\psi = -\frac{2\zeta}{\omega}\rho_m \exp[-2\psi] \tag{5.2}$$

$$\ddot{\psi} + \frac{3\dot{a}}{a}\dot{\psi} - \frac{1}{a^2}\nabla^2\psi = -\frac{2\zeta}{\omega}\rho_m \exp[-2\psi] \tag{5.3}$$

$$\frac{d}{dt}\left(\dot{\psi}a^3\right) - a\nabla^2\psi \quad = \quad N\exp[-2\psi] \tag{5.4}$$

where N is a constant, defined by

$$N \equiv -\frac{2\zeta}{\omega}\rho_m a^3 > 0.$$

We seek a general solution of (5.4) of the form

$$\psi = \psi_h + \delta(\vec{x}, t) \tag{5.5}$$

where $\psi_h(t)$ is the solution to the space-independent problem $(\nabla\psi \equiv 0)$, so by definition $\psi_h(t)$ is an exact solution of

$$\frac{d}{dt}\left(\dot{\psi}_h a^3\right) = N\exp[-2\psi_h]$$

We note immediately an important general property of this equation, that applies to all Friedmann universes with varying α :

No-oscillation theorem: In the BSBM theory, α cannot display oscillatory behaviour in time in a Friedmann universe of any curvature.

The proof is simple: When N is positive (negative) the right-hand side of equation (4.3) is positive (negative), $\psi_h$cannot have an expansion maximum (minimum) since $\dot{\psi}_h = 0$ and $\ddot{\psi}_h < 0$ (> 0) there. Therefore $\psi_h$cannot oscillate in time and so neither can $\alpha = \exp[2\psi]$.

We see that in the case of interest, when $N > 0$, ψ'can have a minimum but thereafter it must always increase irrespective of the behaviour of the expansion scale factor. However, if the equation is linearised in ψ_h this is no longer true if attention is not confined to the small ψ regime where $\exp[-2\psi_h] \approx 1 - 2\psi_h > 0$ and spurious oscillations of ψ (and α) in time can appear to arise at late times if ψ grows. It is of particular interest that this proof that ψ_h cannot have a maximum applies to recollapsing universes $(k = +1)$ as well as to ever-expanding universes $(k \leq 0)$. It also means that oscillations of α with redshift should not be observed in Friedmann universe. This might prove an interesting prediction for future observations to test.

Substituting (5.5) into (5.4) we get

$$\frac{d}{dt}\left(\dot{\delta}a^3\right) - a\nabla^2\delta = N\exp[-2\psi_h]\{\exp[-2\delta] - 1\}$$

So for small δ

$$\frac{d}{dt}\left(\dot{\delta}a^3\right) - a\nabla^2\delta = -2N\delta\exp[-2\psi_h] + O(\delta^2)$$

Now look for separable solutions

$$\delta = T(t)D(\vec{x})$$

and we have

$$\frac{\ddot{T}}{T}a^2 + 3a\dot{a}\frac{\dot{T}}{T} + \frac{2N}{a}\exp[-2\psi_h] = -\mu^2 = \frac{\nabla^2 D}{D} \tag{5.6}$$

where μ^2 is a separation constant with a sign chosen to ensure non-growing, oscillatory, inhomogeneity in $D(\vec{x})$ at spatial infinity . In this equation we can always neglect $2Na^{-1}\exp[-2\psi_h]$ with respect to μ^2 as $t \to \infty$ because ψ_h never falls with time (in the dust era ψ_h grows as $\frac{1}{2}\ln[2N\ln(t)]$ as $t \to \infty$, for example). This is an important feature of the variation of ψ, and α, in BSBM varying-α theories when $\zeta < 0$. It ensures that the kinetic term and the $\zeta_m\exp[-2\psi]$ terms can be neglected in the Friedmann equation asymptotically and the expansion scale factor can self-consistently be assumed to be of the same form as when α does not vary (this is *not* true if $\zeta > 0$). Thus

$$\ddot{T} + \frac{3\dot{a}}{a}\dot{T} + \frac{\mu^2 T}{a^2} = 0 \tag{5.7}$$

and

$$\nabla^2 D = -\mu^2 D$$

so we have the standard separable spherical oscillator solution in spherical polar coordinates:

$$D(r,\theta,\varphi) = \sum_{\ell=0}^{\infty} c_{\mu,\ell} Y_\ell(\theta,\varphi) r^{-1/2} Z_{\ell+\frac{1}{2}}(\mu r)$$

where Z is a cylindrical function and Y the spherical harmonic function. If we specialise to spatially-flat cosmologies with perfect fluid equations of state for pressure p and density ρ of the form

$$p = (\gamma - 1)\rho,$$

then the expansion scale factor will have power law form

$$a = t^n \tag{5.8}$$

with $n = 2/3\gamma$. In these cases we have

$$t\ddot{T} + 3n\dot{T} + \mu^2 T t^{1-2n} = 0 \tag{5.9}$$

Thus, for $n \neq 1$:

$$T(t) = t^{(1-3n)/2}\left[C_1 J_\nu\left(\frac{\mu}{1-n}t^{1-n}\right) + C_2 Y_\nu\left(\frac{\mu}{1-n}t^{2-n}\right)\right], \tag{5.10}$$

$$\nu \equiv \frac{|1-3n|}{2(1-n)} \tag{5.11}$$

while for the curvature-dominated expansion with $n = 1$:

$$T \propto t^q \tag{5.12}$$

$$q = -1 \pm \sqrt{1-\mu^2} \tag{5.13}$$

The late-time behaviour is easily determined as $t \to \infty$:

$$T(t) \propto t^{-n} \times \mathit{oscillations}; \qquad n \neq 1. \tag{5.14}$$

$$T(t) \propto t^{-1+\sqrt{1-\mu^2}}; \qquad n = 1 \tag{5.15}$$

and decays, $T \propto a^{-1}$, as $t \to \infty$. However, as we have already pointed out the oscillatory behaviour is an artefact of the linearisation process and the Bessel-like oscillation are not reached by the solution for ψ.

In the radiation era we can find an solution of eqn. (5.6) for $T(t)$ without neglecting the term $2Na^{-1}\ \exp[-2\psi_h]$ since the radiation universe has the simple exact solution:

$$\psi_h = \frac{1}{2}\log(8N) + \frac{1}{4}\log(t) \tag{5.16}$$

Substituting (5.16) in eqn. (5.6) we find

$$T(t) = \frac{1}{t^{1/4}}\{AJ_m(2\mu t^{1/2}) + BJ_{-m}(2\mu t^{1/2})\}$$

where

$$m = \frac{i\sqrt{3}}{2}$$

and we see explicitly that there is agreement with the asymptote (5.14) of the approximated equation when $n = 1/2$. Similar exact solutions can be found for all universes with $1/3 < n < 2/3$.

The cosmological constant case of $\gamma = 0$ is distinct, with

$$a = \exp[H_0 t]$$

which gives

$$0 = \ddot{T} + 3H_0\dot{T} + \mu^2 T\ \exp[-2H_0 t] \approx \ddot{T} + 3H_0\dot{T}$$

as $t \to \infty$, so

$$T \to T_\infty - \frac{1}{3H_0}\exp[-3H_0(t+t_0)] \to T_\infty$$

This behaviour is in accord with the expectations of a cosmic no hair theorem. It means that if a period of inflation occurs in the very early universe then large scale homogeneity will appear increasingly negligible with time within the event horizon of a geodesically moving observer. In the late stages of a universe like our own, which displays evidence of being accelerated by the presence of a positive cosmological constant, [3], it ensures that time variations in α will not grow. This is to be expected since the inhomogeneities in density are also prevented from growing by the effects of the cosmological constant.

5.3 The case of $\zeta > 0$

When the dark matter is dominated by electric field energy, we have $\zeta > 0$, and the behaviour of eq. (4.3) is very different to that obtained when $\zeta < 0$. Most crucially, the test-motion approximation used above to analyse the behaviour or (4.3) does not apply, even for the purely time-dependent ψ evolution in a Friedmann universe. The solutions obtained for ψ by assuming the scale factor evolution $a = t^n$ of general relativity (with constant α) lead to solutions for ψ (and α) which do not increase with time. For example, we have $\alpha \propto t^{-1}$ in the curvature era and $\alpha \propto \ln(t_0/t)$ in the dust era. These contribute kinetic ($\dot{\psi}^2$) and magnetic contributions ($\zeta \exp[-2\psi]$) terms which dominate the underlying Friedmann equation, (4.2), at large times and the expansion of the universe is not well approximated by that obtained in general relativistic cosmologies with the same equation of state and constant α except over finite non-asymptotic intervals of time. This leads to problems accommodating observational constraints, notably the results of studies of the structure of the microwave background at last scattering [4, 5] and big bang nucleosynthesis [6] in the radiation era because the value of α then is significantly different from today, unlike in the cases of $\zeta < 0$). Cosmologies with $\zeta > 0$ have been discussed in ref. [7] in a theory that is similar in structure to the BSBM theory discussed here. We will discuss the $\zeta > 0$ version of the theory in more detail elsewhere. It is less well behaved and does not seem to provide the smooth and simple perturbation of the standard cosmology with constant α as seen in the negative ζ case.

5.4 Discussion

We have shown that the time-dependent solution to the Friedmann model are stable against the effects of inhomogeneous motions of the ψ field. In the case of inhomogeneous variation the cosmological solutions in universes with scale factor $a(t) =$

t^n, to leading order take the form:

$$\psi(\vec{x},t) = \psi_h(t) + t^{(1-3n)/2}\left[C_1 J_\nu\left(\frac{\mu}{1-n}t^{1-n}\right) + C_2 Y_\nu\left(\frac{\mu}{1-n}t^{2-n}\right)\right]\sum_{\ell=0}^{\infty} c_{\mu,\ell} Y_\ell(\theta,\varphi) r^{-1/2} Z_{\ell+\frac{1}{2}}(\mu$$

when $n \neq 1$, and

$$\psi(\vec{x},t) = \psi_h(t) + t^{-1+\sqrt{1-\mu^2}}\left[A + Bt^{-1-2\sqrt{1-\mu^2}}\right]\sum_{\ell=0}^{\infty} c_{\mu,\ell} Y_\ell(\theta,\varphi) r^{-1/2} Z_{\ell+\frac{1}{2}}(\mu r)$$

when $n = 1$, while for the case of $a(t) = exp[H_0 t]$

$$\psi(\vec{x},t) = \psi_h(t) + O(\exp[-3H_0 t\ ]) \tag{5.17}$$

Thus in all cases we have

$$\psi(\vec{x},t) \to \psi_h(t)$$

as $t \to \infty$ and at late times spatial variations in the fine structure constant decay as

$$\alpha \ = \ \exp[2\psi_h]\{1 + 2t^{-n}\sum_{\ell=0}^{\infty} c_{\mu,\ell} Y_\ell(\theta,\varphi) r^{-1/2} Z_{\ell+\frac{1}{2}}(\mu r) \times oscns + ..\}$$

for $n \neq 1$. Hence, denoting $\alpha_h \equiv \exp[2\psi_h]$, the spatial variation in α decays in time in the $n \neq 1$ universes as

$$\frac{\delta\alpha}{\alpha} \equiv \frac{\alpha - \alpha_h}{\alpha_h} = 2t^{-n}\sum_{\ell=0}^{\infty} c_{\mu,\ell} Y_\ell(\theta,\varphi) r^{-1/2} Z_{\ell+\frac{1}{2}}(\mu r) \times oscns$$

Analogous expressions can be written down mutatis mutandis for $\delta\alpha/\alpha$ in the $n = 0, 1$ cases.

It is important to compare the evolution of the fine structure constant $\alpha(t)$ in the BSBM theory in the homogeneous case with that for the situation admitting inhomogeneous motions of the fine structure 'constant', $\alpha(t, \vec{x})$, here. We have found that inhomogeneity ($\mu \neq 0$) introduces the possibility of damped oscillatory evolution of α with time but this has been shown to be a artefact of the linearisation process. To leading order, the overall pattern of time evolution studied in previous chapters is unaffected by the presence of small inhomogeneities. The spatial variation amplitudes, $\delta\alpha/\alpha$, are found to decay with time as the universe expands and will not be as significant as the overall variation in time of the mean value of $\alpha(t) \propto \ln(t)$ during the dust-dominated phase of a spatially-flat universe. Inhomogeneous test motions of the ψ field will have been decaying in amplitude throughout the period when the universe was dominated by dust if $\zeta < 0$. Therefore we would not expect any significant inhomogeneities to survive at the astronomically interesting epoch $z \sim 1 - 4$ where the value of the fine structure constant can be probed spectroscopically with high precision. However, our discussion has not considered two situations where more significant spatial variations might arise. The first is the situation within gravitationally bound matter inhomogeneities of large scale which separate out from the expansion of the Universe and collapse to form superclusters and clusters of galaxies. These behave in a manner similar to that expected of separate closed universes until deviations from spherical symmetry become significant. Our analysis is not applicable here because the dynamics of the bound inhomogeneities will differ significantly after they separate off from the background expansion. The second situation of interest is that in which perturbations of the Friedmann metric are included in the problem and allowed to couple to spatial variations in ψ, or α. This coupling will lead to small temperature fluctuations in the microwave background radiation. These problems will be discussed in a subsequent analysis.

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Chapter 6

Anthropic arguments from varying constants

6.1 Introduction

The collection of considerations now known as the Anthropic Principles emerged from attempts by Whitrow [1] to understand why it is unsurprising that we find space to have three dimensions, and by Dicke [2] to understand the inevitability of Dirac 'large number' coincidences in cosmology. Dicke recognised that it was unnecessary to introduce the idea of a time-varying gravitational constant in order to understand why we could not fail to observe that the number of protons in the observable universe is of order the square of the ratio of electromagnetic to gravitational force strengths. Subsequently, Dicke inspired a detailed observational and theoretical investigation of gravity theories in which the Newtonian gravitational constant becomes a space-time variable. He was partly motivated by apparent discrepancies between the predictions of standard general relativity and observations of the perihelion precession of Mercury. These discrepancies were subsequently ascribed to errors in the measurements of the shape and diameter of the Sun created by solar surface activity [3].

There have been many investigations of the apparent coincidences that allow complexity to exist in the universe (see [4, 5, 6, 7]). Typically, they examine the stability of life-supporting conditions to small (or large) perturbations to the values of constants of Nature or to quantities fixed by cosmological 'initial' conditions at $t = 0$ or $t = -\infty$. These in turn divide into studies of two sorts: first, those in which the hypothetical changes introduced to the 'constants' are self-consistently permitted by the cosmological or physical theory employed; and second, those in which they are not. An investigation of the first kind might be one in which the cosmological initial conditions were enlarged to allow anisotropies or the possibility of a significant deviation from flatness. An investigation of the second type might note that a change in the observed value of the electron to proton mass ratio to another fixed value would make it difficult to produce ordered molecular structures. Studies of universes in which traditional 'constants' of Nature are changed are restricted by the lack of self-consistent theories which allow all these possible changes to be accommodated. Without them, it is impossible to determine the possible knock-on effects of varying one constant on others.

There are some exceptions. Varying gravitation 'constant', G, (or dimensionless constants formed with it like Gm^2/hc for any mass m), can be studied using scalar-tensor gravity theories [8]. A varying fine structure 'constant' can be studied using the theory of Bekenstein[9] and Sandvik, Barrow and Magueijo (BSBM), laid out in chapters 3 and 4. Moreover, the formulation of physical theories whose true constants inhabit more than three space dimensions provides a framework for the rigorous study of the simultaneous variation of their three-dimensional counterparts [10], [11], [12]. Recently there has also been much interest in theories where a variation in the fine structure constant is due to a change in the light propagation speed[13, 14, 15]. In a later chapter we will propose various methods for experimentally distinguishing between these different theories.

We have shown in chapter 3 that the simplest theory which joins varying α to general relativity via the propagation of a scalar field (BSBM-theory) can explain these observations together with the lack of evidence for a similar level of variation locally, 2 billion years ago, or at very high redshifts, $z \geq 10^3$. In this chapter we will show how this theory also provides some novel anthropic perspectives on the evolution of our universe or others.

There have been several studies, following Carter, [16] and Tryon [17], of the need for life-supporting universes to expand close to the 'flat' Einstein de Sitter trajectory for long periods of time. This ensures that the universe cannot collapse back to high density before galaxies, stars, and biochemical elements can form by gravitational instability, or expand too fast for stars and galaxies to form by gravitational instability (see also [18], [19] and [5]). Likewise, it was pointed out by Barrow and Tipler, [5] that there are similar anthropic restrictions on the magnitude of any cosmological constant, Λ. If it is too large in magnitude it will either precipitate premature collapse back to high density (if $\Lambda < 0$) or prevent the gravitational condensation of any stars and galaxies (if $\Lambda > 0$). Thus existing studies provide anthropic reasons why we can expect to live in an old universe that is neither too far from flatness nor dominated by a much stronger cosmological constant than observed ($|\Lambda| \leq 10\,|\Lambda_{obs}|$).

Inflationary universe models provide a possible theoretical explanation for proximity to flatness but no explanation for the smallness of the cosmological constant. Varying speed of light theories [13, 14, 15, 20] offer possible explanations for proximity to flatness and smallness of a classical cosmological constant (but not necessarily for one induced by vacuum corrections in the early universe). Here, we shall show that if we enlarge our cosmological theory to accommodate variations in some traditional constants then *it appears to be anthropically disadvantageous for a universe to lie too close to flatness or for the cosmological constant to be too close to zero.* This conclusion arises because of the coupling between time-variations in constants like α and the curvature or Λ, which control the expansion of the universe. The onset of a period of Λ or curvature domination has the property of dynamically stabilising the constants, thereby creating favourable conditions for the emergence of structures. This point has been missed in previous studies because they have never combined the issues of Λ and flatness and the issue of the values of constants. By coupling these two types of anthropic considerations we find that too little Λ or curvature can be as poisonous for life as too much.

6.2 Time variation of α

To understand our anthropic discussion we need to sum up the behaviour of the theory laid out in chapters 2 - 4. The Einstein equations are

$$G_{\mu\nu} = 8\pi G\left(T_{\mu\nu}^{matter} + T_{\mu\nu}^{\psi} + T_{\mu\nu}^{em} e^{-2\psi}\right), \tag{6.1}$$

and the ψ field obeys the equation of motion

$$\Box\psi = \frac{2}{\omega} e^{-2\psi} \mathcal{L}_{em}. \tag{6.2}$$

The Friedmann equations are

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\left(\rho_m\left(1 - |\zeta_m| + |\zeta_m| e^{-2\psi}\right) + \rho_r e^{-2\psi} + \frac{\omega}{2}\dot{\psi}^2 + \rho_\Lambda\right) - \frac{k}{a^2}, \tag{6.3}$$

where the cosmological vacuum energy ρ_Λ is a constant that is proportional to the cosmological constant $\Lambda \equiv 8\pi G \rho_\Lambda$. For the scalar field we have

$$\ddot{\psi} + 3H\dot{\psi} = -\frac{2}{\omega} e^{-2\psi} \zeta_m \rho_m \tag{6.4}$$

where $H \equiv \dot{a}/a$, and ζ_m and ω have been defined earlier. In line with previous chapters and the observational data we will again confine ourselves to negative values of ζ_m. The conservation equations give for the non-interacting radiation, and matter densities $\rho_r \propto e^{2\psi} a^{-4}$and $\rho_m \propto a^{-3}$, respectively. This theory enables the cosmological consequences of varying α, to be analysed self-consistently rather than by changing the constant value of α in the standard theory, as in the original proposals made in response to the large numbers coincidences [21].

The cosmological behaviour of the solutions to these equations was studied in chapters 3 and 4 for the $k = 0$ case and is shown in Figure (6.1). The evolution of α is summarised as follows:

1. During the radiation era α is constant and $a(t) \sim t^{1/2}$. It increases in the dust era, where $a(t) \sim t^{2/3}$, until the cosmological constant starts to accelerate the universe, $a(t) \sim \exp[\Lambda t/3]$, after which α asymptotes rapidly to a constant, see fig.(6.1)

2. If we set the cosmological constant equal to zero then, during the dust era, α will increase indefinitely. The increase however, is very slow with a late-time solution for ψ proportional to $\log(2N\log(t))$, see fig.(6.2). N is defined as $N \equiv -2\zeta_m/\rho_m a^3$, a positive constant since we have confined ourselves to $\zeta_m < 0$.

3. If we set the cosmological constant equal to zero and introduce a negative spatial curvature ($k < 0$) then α increases only during the dust-dominated phase, where $a(t) \sim t^{2/3}$, but tends to a constant after the expansion becomes curvature dominated, with $a(t) \sim t$. This case is illustrated in fig.(6.3).

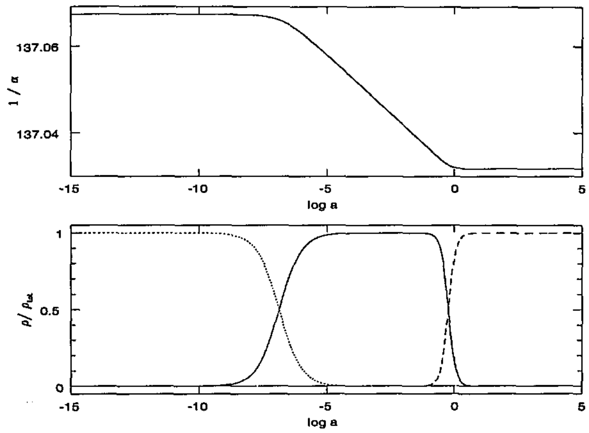

Figure 6.1: The top plot shows the change in alpha throughout the dust epoch ends as lambda takes over the expansion. The lower plot shows the radiation (dotted), dust (solid) and lambda (dashed) densities as fractions of the total energy density.

From these results it is evident that non-zero curvature or cosmological constant brings to an end the increase in the value of α that occurs during the dust-dominated era[1] . Hence, if the spatial curvature and Λ are too small it is possible for the fine structure constant to grow too large for biologically important atoms and nuclei to exist in the universe. There will be a time in the future when α reaches too large a

[1] In some Friedmann universes with initial conditions unlike our own there can be power-law growth of α during the radiation era (see discussion in chapter 4). In such universes the same general effects of negative curvature and positive Λ are seen. They still halt any growth in $\alpha(t)$. Our initial conditions are chosen so as to give a present day value of $\alpha \approx 1/137$. The initial value of alpha would have to be several orders of magnitude lower in order to obtain the power-law growth

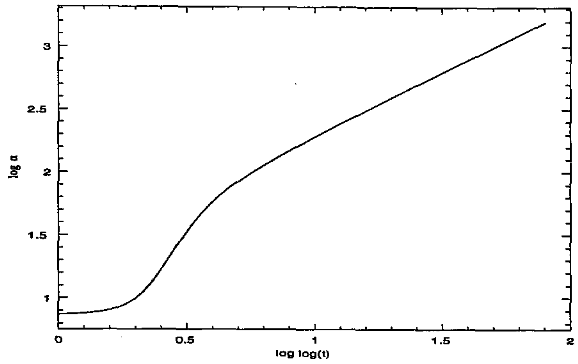

Figure 6.2: $\psi \propto \ln \alpha$ changes as $\log(2N \log t)$ in the dust era.

value for life to emerge or persist. The closer a universe is to flatness or the closer Λ is to zero so the longer the monotonic increase in α will continue, and the more likely it becomes that life will be extinguished. Conversely, a non-zero positive Λ or a non-zero negative curvature will stop the increase of α earlier and allow life to persist for longer. If life can survive into the curvature or Λ-dominated phases of the universe's history then it will not be threatened by the steady cosmological increase in α unless the universe collapses back to high density.

6.3 Anthropic Limits on α

We have seen that varying-α cosmologies with zero curvature and Λ lead to a monotonic increase in α with time. Here we summarise the principal upper limits on α that are needed for atomic complexity and stars to exist. There are a variety of constraints on the maximum value of the fine structure compatible with the existence of nucleons, nuclei, atoms and stars under the assumption that the forms of the laws of Nature remain the same. The running of the fine structure constant with energy due to vacuum polarisation effects leads to an exponential sensitivity of the proton lifetime with respect to the low-energy value of α with $t_{pr} \sim \alpha^{-2} \exp(\alpha^{-1}) m_{pr}^{-1} \sim 10^{32} yrs$. In order that the lifetime be less than the main sequence lifetime of stars we have

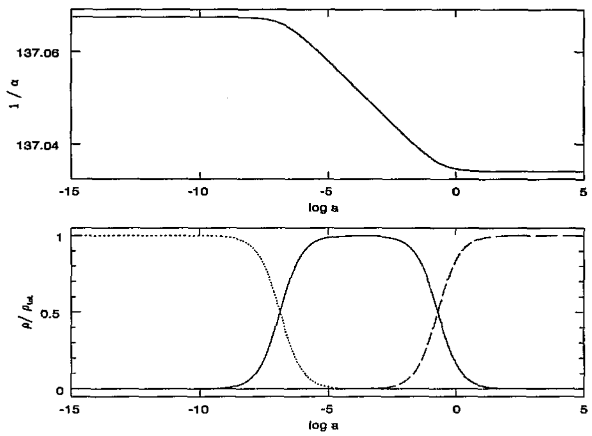

Figure 6.3: Top: The change in alpha comes to an end as curvature takes over the expansion. The bottom graph again shows the different constituents of the universe as a function of the scale factor.

$t_{pr} < (Gm_{pr}^2)^{-1}m_{pr}^{-1}$ which implies that α is bounded above by $\alpha < 1/80$ approximately [22].

The stability of nuclei is controlled by the balance between nuclear binding and electromagnetic surface forces [23]. A nucleus (Z, A) will be stable if $Z^2/A < 49(\alpha_s/0.1)^2(1/137\alpha)$. In order for carbon ($Z = 6$) to be stable we require $\alpha < 16(\alpha_s/0.1)^2$. Detailed investigations of the nucleosynthesis processes in stars have shown that a change in the value of α by 4% shifts the key resonance level energies in the carbon and oxygen nuclei which are needed for the production of a mixture of carbon and oxygen from beryllium plus helium-4 and carbon-12 plus helium-4 reactions in stars [24, 25]. These upper bounds on α are model independent and were considered in more detail in refs. [5], [4] and [6]. However, sharper limits can be found by using our knowledge of the stability of matter derived from analysis of the Schrödinger equation. Stability of matter with Coulomb forces has been proved for non-relativistic dynamics, including arbitrarily large magnetic fields, and for relativistic dynamics without magnetic fields. In both cases stability requires that the fine structure constant be not too large.

The value of α controls atomic stability[2] . If α increases in value then the innermost Bohr orbital contracts and electrons will eventually fall into the nucleus when $\alpha > Z^{-1}m_{pr}/m_e$. As α increases, atoms all become relativistic and unstable to pair production. In order that the electromagnetic repulsion between protons does not exceed nuclear strong binding $e^2/r_n < \alpha m_\pi$ is needed and so we require $\alpha < 1/20$. It is also known that atomic instability of atoms with atomic number Z occurs in the relativistic Schrödinger equation when the fine structure constant is increased in value to $\alpha = \frac{2}{\pi Z}$. However, when the many-electron and many-nucleon problem is examined with the relativistic Schrödinger theory there is a bound on α for stability that is independent of

[2] Note that if the electron mass and velocity of light are varied along with the value of α then the eigenvalues of the non-relativistic Schrödinger equation can remain invariant and atomic structure is unchanged [5]. Here, we break the scale invariance by varying only α. Note that the invariance does not hold for the relativistic case[26]

Z [27]. If $\alpha < 1/94$ then stability occurs all the way up to the critical value $\alpha = \frac{2}{\pi Z}$, whereas if $\alpha > 128/15\pi$ the 'atomic' system is unstable for all values of Z. In the presence of arbitrarily large magnetic fields, which aid binding by creating a two-dimensional form for the potential, matter composed of electrons and nuclei is known to be unstable if α or Z is too large: matter is stable if $\alpha < 0.06$ and $\alpha < 0.026(6/Z)^{1/2}$,[28],[29].

If stars are to exist, their centres must be hot enough for thermonuclear reactions to occur. This requires α to be bounded above by $\alpha^2 < 20m_e/m_{pr}$. Carter has also pointed out the existence of a very sensitive condition $\alpha^{12} \sim (m_e/m_{pr})^4 \, Gm_{pr}^2$,that must be met if stars are to undergo a convective phase, although this stringent condition no longer seems to be essential for planetary formation [16].

The results collected above show that there are a number of general *upper limits* on the value of α if atoms, molecules, and biochemistry are to exist. These bounds do not involve the gravitation constant explicitly. Other astrophysical upper bounds on α exist in order that stars be able to form but these involve the gravitational constant.

6.4 Time variation of G

A similar trend can be found in relativistic cosmologies in scalar-tensor gravity theories. Consider the paradigmatic case of Brans-Dicke (BD) theory to fix ideas. The form of the general solutions to the Friedmann metric in BD theories are fully understood [30],[31]. The general solutions begin at high density dominated by the BD scalar field $\phi \sim G^{-1}$ and approximated by the vacuum solution. At late times they approach particular exact power-law solutions for $a(t)$ and $\phi(t)$ and the evolution is 'Machian' in the sense that the cosmological evolution is driven by the matter content rather than by the kinetic energy of the free ϕ field. There are three essential field equations for the evolution of ϕ and $a(t)$ in a BD universe

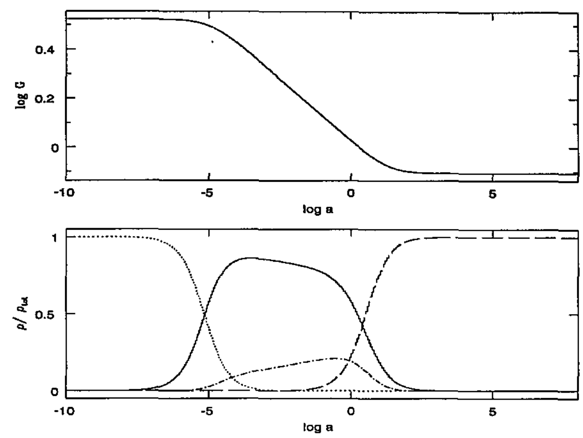

Figure 6.4: Top plot shows cosmological evolution of Brans-Dicke theory, with $\omega = 10$, from radiation domination into dust domination and through to curvature driven expansion. Lower plot shows radiation (dotted) , dust (solid) and curvature (dashed) energies, as well as the scalar field energy (combined), as a fraction of the total energy density.

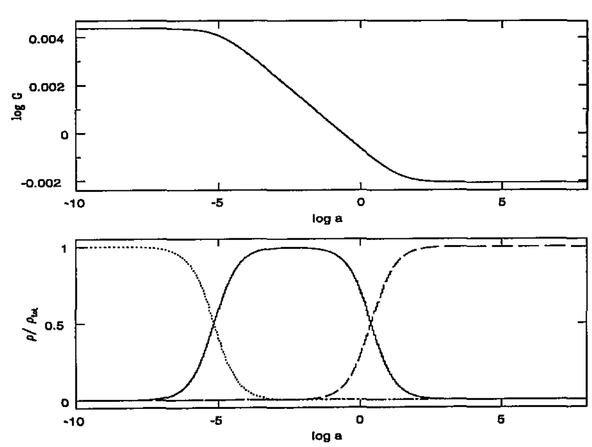

Figure 6.5: Similar evolution of Brans-Dicke theory with $\omega = 1000$.

$$
\begin{aligned}
3\frac{\dot{a}^2}{a^2} &= \frac{8\pi\rho}{\phi} - 3\frac{\dot{a}}{a}\frac{\dot{\phi}}{\phi} + \frac{\omega_{BD}}{2}\frac{\dot{\phi}^2}{\phi^2} - \frac{k}{a^2} \\
\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} &= \frac{8\pi}{3+2\omega}(\rho - 3p) \\
\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + p) &= 0
\end{aligned}
$$

Here, ω_{BD} is the BD constant parameter and the theory reduces to general relativity in the limit $\omega_{BD} \to \infty$ and $\phi = G^{-1} \to$constant.

In the radiation era the scale factor approaches the standard general relativistic behaviour for large times:

$$a(t) \sim t^{1/2}; \qquad G = constant \tag{6.5}$$

After the dust density dominates the dynamics the expansion approaches a simple exact solution with

$$a(t) \propto t^{(2-n)/3}; \qquad G \propto t^{-n}, \tag{6.6}$$

which continues until the curvature term takes over the expansion. Here, n is related to the constant Brans-Dicke ω_{BD} parameter by

$$n \equiv \frac{2}{4 + 3\omega_{BD}} \tag{6.7}$$

and the usual general relativistic Einstein de Sitter universe is obtained as $\omega_{BD} \to \infty$ and $n \to 0$. If the universe is open, $(k = -1)$, then the negative curvature will eventually dominate the gravitational effects of the dust and then the BD model approaches the general relativistic Milne model with constant G

$$a(t) \propto t; \qquad G = constant \tag{6.8}$$

Again, we see the same pattern of behaviour seen for the evolution of α in the BSBM theory. The smaller the curvature term, so the longer the dust-dominated era lasts, and the greater the fall in the value of G, and the smaller its ultimate asymptotic value when the curvature intervenes to turn off the variation. In general, in such cosmologies, if there exists a critical value of G below which living complexity cannot be sustained, then a universe that is too close to flatness will have a smaller interval of cosmic history during which it can support life.

So far, we have discussed only the independent variation of α and G. What happens if they both vary at the same time? Previous studies of varying constants have only examined the time-variation of a single 'constant'. In chapter 8 we will present a unified theory, which incorporates the both BSBM varying α and BD varying G theories discussed above. When both α and G are allowed to vary simultaneously in this theory we find that our general conclusions still hold (see our discussion in chapter 8), although the quantitative details are changed. During the dust era of a flat Friedmann universe with varying $\alpha(t)$ and $G(t)$,their time-evolution approaches an attractor in which the product αG is a constant and

$$\alpha \propto G^{-1} \propto t^{n} \tag{6.9}$$

where n is given by eq. (6.7). Thus we see that the G evolution is left unchanged by the effects of varying α, but variation of G changes the time evolution of $\alpha(t)$ from a logarithm to a power-law in time. As before, the longer the dust era lasts before it is ended by deviation from flatness or zero cosmological constant, the longer the time-increase of α continues, inevitably leading to values that make any atom-based complexity impossible.

6.5 Discussion

We have shown that some theories which include the time variation of traditional constants like α and G introduce significant new anthropic considerations. A theory which self-consistently introduces the space-time variation of a traditional constant scalar quantity is strongly constrained in form by the requirements of causality and second-order propagation equations [9]. Typically, this requirement leads to equations for the driving scalar, φ that have the form $\Box\varphi$ proportional to linear combinations of the energy-momentum components. Explicit examples are provided by the Bekenstein-Sandvik-Barrow-Magueijo and Brans-Dicke theories. This structure ensures that the evolution of the 'constant' whose variations are derived from those of φ is strongly dependent upon the material or geometrical source governing the background expansion dynamics. In the case of varying α we have shown in our discussions in chapters 3 and 4 that this ties the epoch after which time-variations in α become very small to the time when the cosmological constant starts to accelerate the expansion of the universe. In these theories there is therefore the possibility of a habitable time zone of finite duration during which a constant like α or G falls within a biologically acceptable range.

Surprisingly, there has been almost no consideration of habitability in cosmologies with time-varying constants since Haldane's discussions [32] of the biological consequences of Milne's bimetric theory of gravity with two timescales, one for atomic phenomena, another for gravitational phenomena [33]. Since then attention has focussed upon the consequences of universes in which the constants are different but still constants. Those cosmologies with varying constants that have been studied have not considered the effects of curvature or Λ domination on the variation of constants and have generally considered power-law variation to hold for all times. The examples described here show that this restriction has prevented a full appreciation of the coupling between the expansion dynamics of the universe and the values of the constants that define the course of local physical processes within it. Our discussion of a theory with varying α shows for the first time a possible reason why the 3-curvature of universes and the value of any cosmological constant may need to be bounded *below* in order that the universe permit atomic life to exist for a significant period. Previous anthropic arguments have shown that the spatial curvature of the universe and the value of the cosmological constant must be bounded *above* in order for life-supporting environments (stars) to develop. We note that the lower bounds discussed here are more fundamental than these upper bounds because they derive from changes in α which have direct consequences for biochemistry whereas the upper bounds just constrain the formation of astrophysical environments by gravitational instability (for alternative scenarios see ref. [34]). Taken together, these arguments suggest that within an ensemble of all possible worlds where α and G are time variables, there might only be a finite interval of *non-zero* values of the curvature and cosmological constant contributions to the dynamics that both allow galaxies and stars to form and their biochemical products to persist.

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[26] This is easy to see from the energy levels of the Hydrogen atom. In the non-relativistic case we have $E_n = -\frac{1}{2}mc^2\frac{Z^2\alpha^2}{n^2}$, so the invariance in changing m, c or α is obvious. However, this fails when the relativistic corrections of higher powers in α are added, e.g. $E_n = -\frac{1}{2}mc^2\frac{Z^2\alpha^2}{n^2} - \frac{mc^2}{2n^4}\left(\frac{n}{J+1/2} - \frac{3}{4}\right)\alpha^4 + \cdots$.

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Chapter 7

Experimental tests for distinguishing different varying-α theories

7.1 Introduction

The observations of Webb et. al.[1, 2, 3] raise a question which we sofar have not attempted to answer: which of e, $\hbar$ and c might be responsible for any observed change in α and what operational meaning should be attributed to such a determination? Undoubtedly, in the sense of [4], one has to make an operationally "meaningless" choice of which dimensional constant is to become a dynamical variable (see however refs. [5] and [6] for a recent debate on this issue). Yet, we suggest that in practice this choice is never arbitrary; it should be clearly dictated by simplicity once the detailed dynamics of the theory have been established. Here, we argue that the dynamics have observational implications: a combination of experiment and simplicity therefore selects one member of a dimensionless combination (α) of dimensional constants (e, $\hbar$ and c) to which we should preferentially ascribe its space-time variation. In this chapter we will present a number of clear experimental tests which could distinguish rival theories of α variation which are expressed through explicit change in e or c. Existing theories will be used as examples.

Several theoretical contexts for the Webb et al results have been explored. In previous chapters we have laid out the framework for a varying electric charge theory (which we shall denote BSBM), inspired by an earlier construction of Bekenstein

[7]. This model is simple[1] in the sense that it is a minimal extension to general relativity, not breaking with any fundamental principles such as causality and Lorentz invariance, and with only one scalar field added, coupling only to electromagnetism. No potential for the field is needed, and the only free parameter in the model is an energy scale not far from Planck scale. A supersymmetric version of this theory was created in [8]. Phenomenological models expanding α around its present value have pointed out the possibility of identifying the scalar field driving the variation in α with a quintessence field[9, 10], possibly constraining the possible classes of quintessence potentials. Chiba[10] also identified the difficulty in reconciling the Oklo data with the Quasar results. Carroll [11] suggests that an approximate global symmetry could suppress the coupling of the quintessence field to parts of the Standard Model other than electromagnetism. This would still allow a finely tuned quintessence field to drive a change in α. Various dilatonic alternatives, in which all coupling constants vary as a function of a single field, may also be considered (including dilaton couplings to the cosmological constant [8]). Other candidates to explain variations in α are the so-called varying speed of light (VSL) theories [12, 13, 14, 15, 16, 17, 18], which also offer an alternative to inflation for solving cosmological problems. Although VSL theories generally entail breaking Lorentz invariance it has been shown[17, 18] that this is not necessarily the case. Ref. [18] present a class of two-metric VSL cosmologies compatible with both classical Einstein gravity and low-energy particle physics. In this framework there is a second separate metric to which photons couple, distinct from the spacetime metric which describes the gravitational field and which couples to ordinary matter. These models also solve most cosmological puzzles usually solved by inflation, but since there is no violation of the strong energy condition, they do not explain the flatness problem.

[1] Indeed there are ambiguities in labelling theories 'simple' or 'natural'. The simplicity could equally well lie in a theory being derived from a fundamental theory with particularly attractive higher symmetries without necessarily being a minimal extension to general relativity (GR). There are also more than one direction in which one can depart from GR

As examples of varying e and varying c theories we take the BSBM theory and the VSL theory presented in [17]. By introducing an appropriate change of units we can turn VSL into a constant c theory, but the dynamics will then look unnecessarily complicated; likewise BSBM can be rephrased as a constant e, varying c theory, with a concomitant increase in complexity. This is why we say that BSBM is a varying e theory while the theory in [17] is a VSL theory: dynamics fixes the choice. Crucially, the dynamics also have unambiguous observational implications. We will show that with standard dark matter this VSL predicts an *increasing* α, as a function of cosmological time[2] . By contrast, BSBM predicts a *decreasing* α, a conclusion which can only be reversed by a different choice of dark matter composition, as explained in chapter 3. This is a striking difference, but pending the determination of the nature of the dark matter one can use both BSBM and VSL to fit the Webb et al results[19]. The same remark applies to other cosmological tests, such as constraints arising from the cosmic microwave background (CMB) and Big Bang Nucleosynthesis (BBN) [20, 21].

However, BSBM and VSL theories also make different predictions regarding *spatial* variations in α near massive objects. Due to these variations all changing-α theories predict a 'fifth force' effect [7, 8, 22, 19], but we will see that the exact details can distinguish between BSBM and VSL. In BSBM theory the fifth force induces an anomalous acceleration which, unlike gravity, depends on the material composition of the test particle and so violates the weak equivalence principle (WEP). This VSL theory and others, on the other hand, are consistent with the WEP, as first noted by Moffat[19].

The exact level of WEP violation predicted by BSBM depends upon an unsolved problem in nuclear and hadronic physics: how much of the mass-energy of nuclei is of electrostatic nature? As yet, there is no reliable answer to this question [23] but we can still estimate the magnitude of WEP violation, which reveals that the BSBM theory is

[2] This is not unique to the theory in question; the VSL theory of [18] can explain both increasing and decreasing α

marginally consistent with current Eötvös experiments. However, the next generation of WEP tests, such as the STEP project [24], will easily be sensitive enough to detect violations of the WEP as predicted by BSBM even by the most conservative estimates. Should violations be observed, it should be seen as a success for varying e theories. If not, then we must narrow our interest to VSL theories in order to accommodate observational signals of varying α. Thus, space experiments such as STEP can provide an independent experimental test of any astronomical evidence for varying α, and decide between a varying e or c interpretation.

7.2 The BSBM and VSL frameworks

We start by comparing the two theories to be used as exemplars. In the BSBM varying α theory, the quantities c and $\hbar$ are taken to be constant, while e varies as a function of a real scalar field ψ, with $e = e_0 e^{\psi}$. As shown in chapters 2 and 3, it is possible to rewrite this theory in such a way that ψ only couples to the free electromagnetic lagrangian $\mathcal{L}_{em}$. The field tensor $f_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu$ and the covariant derivatives $D_\mu = \partial_\mu + ie_0 a_\mu$ then do not contain ψ, and the action takes the form:

$$S = \int d^4x\sqrt{-g}\left(\mathcal{L}_g + \mathcal{L}_{mat} + \mathcal{L}_\psi + \mathcal{L}_{em}e^{-2\psi}\right), \tag{7.1}$$

where $\mathcal{L}_\psi = -\frac{\omega}{2}\partial_\mu\psi\partial^\mu\psi$, $\mathcal{L}_{em} = -\frac{1}{4}f_{\mu\nu}f^{\mu\nu}$, and $\mathcal{L}_{mat}$ (the lagrangian of all matter fields apart from $\mathcal{L}_{em}$) does not depend on ψ. The gravitational lagrangian is the usual $\mathcal{L}_g = \frac{1}{16\pi G}R$, with R the curvature scalar.

In contrast, the covariant VSL theory proposed in [17] assumes that c varies, and builds the simplest dynamics on this premise, which is equivalent to a choice of a system of units. It assumes that $c = c_0 e^{\chi}$ (with χ another real scalar field) and that the *full* matter Lagrangian $\mathcal{L}_M$ does not contain χ. Up to a free parameter, q, this assumption fixes how all matter couplings scale with c; in particular, one has for all interactions i

associated with gauge charges e_i that $\alpha_i \propto e_i \propto \hbar c \propto c^q$. The action is [25]

$$S = \int d^4x\sqrt{-g}\left(\mathcal{L}_g + \mathcal{L}_\chi + \mathcal{L}_M e^{b\chi}\right), \tag{7.2}$$

with $\mathcal{L}_\chi = -\frac{\omega}{2}\partial_\mu\chi\partial^\mu\chi$, and $\mathcal{L}_g$ is as given above. It was shown in [17] that only when $b + q \neq 0$ can these theories be conformally mapped into dilaton theories, and into Brans-Dicke theories only when $q = 0$. This theory has an obvious novelty when compared to BSBM: all α_i are variable. However, recent cosmological variations in non-electromagnetic α_i are beyond the reach of current direct astrophysical observations. Hence for the purpose of this Letter we shall ignore their consequences.

7.3 Cosmological comparisons

Varying the action with respect to the metric leads to straightforward generalizations of Einstein's equations, see ref. [17] and chapter 3. Variation with respect to the new scalar fields leads to dynamical equations for α. For small variations, $\delta\alpha/\alpha \ll 1$, these are:

$$\Box\frac{\delta\alpha}{\alpha} = \frac{4}{\omega}\mathcal{L}_{em} \tag{7.3}$$

for BSBM, and

$$\Box\frac{\delta\alpha}{\alpha} = -\frac{bq}{\omega}\mathcal{L}_M \tag{7.4}$$

for VSL. In both cases the right-hand side is zero for relativistic matter, predicting negligible variations in α during the radiation-dominated cosmological epoch. Two striking differences appear in the matter epoch, when the RHS becomes non-negligible, in both the coupling parameters and the driving source $\mathcal{L}$. The requirement that the fields χ and ψ have a positive definite energy forces $\omega > 0$. This fixes the sign of the coupling for BSBM $(4/\omega)$ but not for VSL $(-bq/\omega)$. The source $\mathcal{L}$ is also different for each theory and is parameterized by different ratios determined by the dark matter: $\zeta = \mathcal{L}_{em}/\rho$ for BSBM, and $\xi = \mathcal{L}_M/\rho$ for VSL.

The value of ζ for baryonic and dark matter has been disputed [22, 8] (see discussion in chapter 3 as well). It is the difference between the percentage of mass in electrostatic and magnetostatic forms. As explained in chapter 3, we can at most *estimate* this quantity for neutrons and protons, with $\zeta_n \approx \zeta_p \sim 10^{-4}$. We may expect that for baryonic matter $\zeta \sim 10^{-4}$, with composition-dependent variations of the same order. The value of ζ for the dark matter, for all we know, could be anything between -1 and 1. Superconducting cosmic strings, or magnetic monopoles, display a *negative* ζ, unlike more conventional dark matter. On the other hand it was argued in [17] that the value of ξ (characterizing the VSL dynamics in the matter epoch) is -1 for all non-relativistic matter. This is equivalent to requiring that non-relativistic matter is dominated by its potential energy (including rest mass) rather than by its kinetic energy T. We shall use this fact in the rest of the chapter although it is not essential for most of what follows.

It is clear that the only way to obtain a cosmologically increasing α in BSBM is with $\zeta < 0$, i.e with unusual dark matter, in which magnetic energy dominates over electrostatic energy. In chapter 3 we showed that fitting the Webb et al results requires $\zeta_m/\omega = -2 \pm 1 \times 10^{-4}$, where ζ_m is weighted by the necessary fractions of dark and baryonic matter. On the other hand VSL theory fits the Webb et al results with $bq/\omega = -8 \times 10^{-4}$, for all types of dark matter. Hence, if we were to determine that $\zeta > 0$ for the dark matter in the universe, we could experimentally rule out BSBM but not VSL. This is just one way in which the question in the title of this chapter could be answered. However, pending identification of the dark matter, we may still answer this question by looking at spatial variations in α.

7.4 Spatial comparisons

In all causal varying-α theories defined by a wave equation the observed redshift dependence of α requires there also to be spatial variations near compact massive bodies (see chapter 3 and ref. [29]). The relevant equations may be obtained by dropping the time dependence in (7.3) and (7.4). Then, a linearized spherically symmetric solution in the vicinity of an object with mass M_s and $\zeta = \zeta_s$ is

$$\frac{\delta\alpha}{\alpha} = -\frac{\zeta_s}{\omega}\frac{M_s}{\pi r} \approx 2 \times 10^{-4}\frac{\zeta_s}{\zeta_m}\frac{M_s}{\pi r} \tag{7.5}$$

for BSBM

$$\frac{\delta\alpha}{\alpha} = -\frac{bq}{\omega}\frac{M_s}{4\pi r} \approx 2 \times 10^{-4}\frac{M_s}{\pi r} \; . \tag{7.6}$$

for VSL. We note that the level of spatial variations in BSBM, given [2, 3], depends on the nature of the dark matter (the ratio ζ_s/ζ_m), whereas for VSL it does not. In VSL, α *increases* near compact objects (with decreasing c if $q < 0$, with increasing c if $q > 0$) but in BSBM α *decreases* (since $\zeta_m < 0$ and $\zeta_s > 0$). In VSL theories, near a black hole α could become much larger than 1, so that electromagnetism would become non-perturbative with dramatic consequences for the physics of black holes. In BSBM precisely the opposite happens: electromagnetism switches off.

Spatial variations lead to a number of observable effects which sharply distinguish between VSL and BSBM. Most obviously α could be measured in absorption lines from compact objects, as explained in [17] for VSL and chapter 3 for BSBM. More subtly, alpha gradients induce a 'fifth force' effect. In order to compute this force one must model ζ or ξ for test bodies. In BSBM the test-particle lagrangian may be split as $\mathcal{L}_t = \mathcal{L}_m + e^{-2\psi}\mathcal{L}_{em}$. Variation with respect to the metric leads to a similar split of the stress-energy tensor, producing an energy density of the form $\rho((1-\zeta_t) + \zeta_t e^{-2\psi})$, and so a mass of $m((1-\zeta_t) + \zeta_t e^{-2\psi})$, (assuming electric fields dominate). In order to preserve their ratios of $\zeta_t = \mathcal{L}_{em}/\rho$ test particles may thus be represented by

$$\mathcal{L}(y) = -\int d\tau \; m((1-\zeta_t) - \zeta_t e^{-2\psi})[-g_{\mu\nu}\dot{x}^\mu \dot{x}^\nu]^{\frac{1}{2}} \frac{\delta(x-y)}{\sqrt{-g}} \tag{7.7}$$

where over-dots are derivatives with respect to the proper time τ. This leads to equations of motion:

$$\ddot{x}^\mu + \Gamma^\mu_{\alpha\beta}\dot{x}^\alpha\dot{x}^\beta + \frac{2\zeta_t e^{-2\psi}}{(1-\zeta_t) - \zeta_t e^{-2\psi}}\partial^\mu\psi = 0 \tag{7.8}$$

which in the non-relativistic limit (with $\zeta_t \ll 1$) reduce to

$$\frac{d^2x^i}{dt^2} = -\nabla_i\phi - 2\zeta_t\nabla_i\psi\ , \tag{7.9}$$

where ϕ is the gravitational potential. Thus we predict an anomalous acceleration:

$$a = \frac{M_s}{r^2}\left(1 + \frac{\zeta_s\zeta_t}{\omega\pi}\right) \tag{7.10}$$

Violations of the WEP occur because ζ_t is substance dependent. For two test bodies with ζ_1 and ζ_2 the Eötvös parameter is:

$$\eta \equiv \frac{2|a_1 - a_2|}{a_1 + a_2} = \frac{\zeta_s|\zeta_1 - \zeta_2|}{\omega\pi}. \tag{7.11}$$

This can be written more conveniently as the product of the following 3 factors:

$$\eta = \left(\frac{\zeta_E|\zeta_1 - \zeta_2|}{\pi\zeta_p}\right)\left(\frac{\zeta_p}{\zeta_m}\right)\left(\frac{\zeta_m}{\omega}\right). \tag{7.12}$$

The last factor is the coupling that determines cosmological time variations in α, and using the results[2, 3] is best fitted to be $\zeta_m/\omega \approx -10^{-4}$. If we take $\zeta_n \approx \zeta_p \approx |\zeta_p - \zeta_n| = \mathcal{O}(10^{-4})$ then for typical substances the first factor is $\approx 10^{-5}$. Hence, we need $\zeta_m = \mathcal{O}(1)$ to produce $\eta = \mathcal{O}(10^{-13})$,just an order of magnitude below existing experimental bounds.

In contrast to this VSL theories predict that for all test particles

$$\mathcal{L}(y) = -\int d\tau\ m e^{b\chi}[-g_{\mu\nu}\dot{x}^\mu\dot{x}^\nu]^{\frac{1}{2}}\frac{\delta(x-y)}{\sqrt{-g}}, \tag{7.13}$$

where we have assumed $\xi = -1$. This leads to an anomalous acceleration of equal magnitude for all test particles, so that there are no WEP violations. This new acceleration does imply corrections to the standard tests of general relativity, such as the precession of Mercury's perihelion, light deflection and radar echo time-delay [26, 29]. These were studied in [29] and impose the undemanding constraint of $b^2/\omega < 10^{-2}$ [30]. Therefore we conclude that an increase of about an order of magnitude in the experimental sensitivity to non-zero η would decide between the BSBM and VSL theories.

Webb et al [1, 2, 3] caution that their results might be due to some uninvestigated systematic effect. For this reason it is important to seek independent observational verification. Direct measurement of WEP violations at the predicted level could be seen as a direct confirmation of the source of the astronomical results. Spatial fluctuations in α could also be directly mapped from spectroscopy of lines formed in very compact objects or their accretion disks (see our discussion in chapter 3 and ref. [29]). But more realistically we note that Earth-based atomic-clock experiments could also measure these fluctuations. Atomic clocks tick at a rate $\tau^{-1} \propto (E_e \alpha^2)/\hbar$, where E_e is the electron rest energy. Hence atomic-clock experiments able to measure gravitational redshifts will suffer from an extra effect: in BSBM theories these clocks tick slower in gravitational wells, with $\tau \propto 1/\alpha^2$, whereas in VSL $\tau \propto 1/c^{2q+1} \propto 1/\alpha^{2+1/q}$. Any hyperbolic varying-α theory explaining [2, 3] should predict a similar effect.

In general, any gravitational-redshift experiment should be sensitive to a varying α. One example is the Pound-Rebka-Snyder experiment, which lets a γ-ray photon emitted by a ^{57}Fe crystal fall in a gravitational field only to observe its resonant absorption by a ^{57}Fe target. This resonant absorption is made possible by the Mössbauer effect, in which the recoil momentum on emission and absorption is taken up by the whole crystal, so that essentially no energy is lost on emission and absorption. The effect has been used to observe gravitational redshifts, but the emitted photon's energy also depends upon α. For small variations in α the energy shift is $\delta E/E = C\delta\alpha/\alpha$ with C of order 1 (but not very well known). A similar effect will occur in experiments using Rydberg lines, with a shift in wavelength $\delta\lambda/\lambda = -2(\delta\alpha/\alpha)$ (for both VSL and BSBM theories). Once the photon is emitted varying-α theories predict no extra redshift for free-flying photons (since $\mathcal{L} = 0$ for photons). However, the observed gravitational redshift of frequencies takes the form $\delta\nu/\nu = (1 + \alpha_{PPN})\delta\phi$, with a non-zero PPN parameter α_{PPN} induced *at emission*. Using (7.5) we find that for BSBM theory $\alpha_{PPN} = 2\zeta_s/(\pi\omega)$, with the quasar data [2, 3] then implying that $\alpha_{PPN} \approx 10^{-8}$. For

VSL theory care must be taken, because $\delta\lambda/\lambda$, $\delta\nu/\nu$ and $\delta E/E$ are distinct quantities. Defining α_{PPN} in terms of frequency in the conventional way and using (7.6) we have that $\alpha_{PPN} \approx (2+q^{-1})bq/(4\pi\omega) \approx -(2+q^{-1})10^{-5}$. Hence BSBM theory predicts a stronger redshift than general relativity, with corrections of order 10^{-8}. If $q \ll 1$, VSL theory predicts a weaker redshift effect with corrections of order 10^{-5}; but this conclusion is changed if $q \approx -1/2$. Both BSBM and VSL theories are consistent with the current bound of $\alpha_{PPN} < 10^{-4}$ [26]. Any causal varying- α theory should predict a non-zero correction to the relativistic redshift formulae.

7.5 Discussion

In summary, we have in this chapter explained how a combination of experiment and common sense may distinguish a varying c from a varying e. Using only minimal versions of such theories, we have shown how they can be distinguished by weak equivalence principle violations, by the type of dark matter required to give the variations inferred from quasar observations[2, 3], and by gravitational-redshift experiments. In non-minimal varying-e and -c theories, the distinguishing observational signatures should be even more obvious. For instance, if Lorentz invariance were found to be broken, [27, 28], then a varying-c theory would provide a better framework for expressing variations in α.

Finally, we note that the experiments proposed in this chapter are by no means the only discriminators between varying-e and -c expressions of a varying α. In [31] the authors examined black hole thermodynamics, by changing the values of e and c in their description of black hole thermodynamics (which, however, may be too simplistic [29]). In this context they found that interpreting a varying α as varying e or c leads to opposite black-hole dynamics, with a varying-e contradicting the second law of thermodynamics. In principle one could test whether or not black hole areas always increases in time in the next generation of gravitational-wave observatories. Like the various experiments described in this chapter, this is experimentally unambiguous, since the ratio of two areas is dimensionless.

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Chapter 8

Simultaneously Varying α and G

8.1 Introduction

There have been many studies of the cosmological consequences of allowing some of the traditional constants of Nature to change. These include evaluations of the effects of altering the observed value of a constant to another constant value and studies of the time-evolution of 'constants' in generalisations of the general theory of relativity that allow them to become space-time variables. The most studied case is that of varying the Newtonian gravitation constant, G, through the Brans-Dicke (BD) scalar-tensor theory of gravity [1]. In the earlier chapters of this thesis, following Bekenstein, [2], we have developed a theory (BSBM) which describes the space-time variation of the fine structure constant. In these, and other, studies of varying constants only a single constant is allowed to vary at one time. However, since we have no understanding of why the constants of Nature take the values that they do, whether they are logically independent, or even whether they all are truly constant, this restriction is somewhat artificial. Motivated by recent observational evidence for a time evolution of the fine structure 'constant', α, at redshifts $z \sim 1 - 3.5$, [3], [4], [5], we have unified the BD and BSBM theories to produce an exact theory which describes the simultaneous variation of α and G. This type of model also provides a framework within which to consider the consequences of changes in the scale of extra dimensions of space on apparent three-dimensional coupling constants.

In section 2 we set up the theory and evolution equations for Friedmann universes in a theory that generalises general relativity to include varying α and G. In section 3 we show how to find the cosmological solutions during the dust-dominated eras. We find an exact solution where αG is constant during the dust era while α and G^{-1} both increase with time. We then determine analytically the coupled evolution of α and G during the radiation, curvature, and vacuum-energy dominated eras of cosmological expansion. From here we go on to check the solutions numerically and we show how in Universes like our own, with actual initial values for α and G the asymptotic behaviour is never reached. Instead we find constant α and G in the radiation era, slow growth of α and slow decrease in G in the dust epoch, constant values for both in curvature dominated universe, and constant α and decreasing G in Λ dominated epoch. Generally we find that the overall evolution of the expansion scale factor of the universe is dictated by the G variation and assumes the form found in the Brans-Dicke theory to a very good approximation irrespective of the α variation. The evolution of α is influenced by the G variation but does not differ much from that found in the BSBM cosmologies where only α varies.

8.2 Field Equations

We introduce the structure of the BSBM theory for varying α as another matter field in Brans-Dicke theory. The resulting theories has two scalar fields: the BD field ϕ propagating variations in G, and the field ψ propagating variations in α. The action for this theory becomes

$$S = \int d^4x\sqrt{-g}\left(R\phi + \frac{16\pi}{c^4}\mathcal{L} - \omega_{BD}\frac{\phi_{,\mu}\phi^{,\mu}}{\phi}\right) \tag{8.1}$$

where

$$\mathcal{L} = \mathcal{L}_m + \mathcal{L}_{em}\exp(-2\psi) + \mathcal{L}_\psi, \tag{8.2}$$

and

$$\mathcal{L}_\psi = -\frac{\omega}{2}\psi_{,\mu}\psi^{,\mu}.$$

The field equations for the theory, specialised to the case of a homogeneous and isotropic Friedmann space-time metric containing dust and radiation perfect fluids are:

$$
\begin{aligned}
3\frac{\dot{a}^2}{a^2} &= \frac{8\pi}{\phi}\left(\rho_m(1-|\zeta|+|\zeta|e^{-2\psi})+\rho_r e^{-2\psi}+\rho_\psi\right)-3\frac{\dot{a}}{a}\frac{\dot{\phi}}{\phi}+\frac{\omega_{BD}}{2}\frac{\dot{\phi}^2}{\phi^2}-\frac{k}{a^2} && (8.3)\\
\ddot{\phi}+3\frac{\dot{a}}{a}\dot{\phi} &= \frac{8\pi}{3+2\omega_{BD}}(\rho_m-2\rho_\psi) && (8.4)\\
\ddot{\psi}+3H\dot{\psi} &= -\frac{2}{\omega}\exp(-2\psi)\zeta\rho_m && (8.5)\\
\dot{\rho_m}+3H\rho_m &= 0 && (8.6)\\
\dot{\rho_r}+4H\rho_r &= 2\dot{\psi}\rho_r && (8.7)
\end{aligned}
$$

where $\rho_\psi = \frac{\omega}{2}\dot{\psi}^2$ is the kinetic energy density for the ψ fluid, with ω the coupling setting the relevant energy scale for the ψ-field. ζ is defined as the ratio $\mathcal{L}_{em}/\rho_m$ averaged over all types of matter in the universe. The fine structure 'constant' is given by ($\hbar = c = 1$)

$$\alpha \equiv \alpha_0 \exp(2\psi), \tag{8.8}$$

where α_0 is the present day value of the fine structure 'constant'. The present-day value of G is set equal to unity.

We shall confine our attention to the case with $\zeta < 0$ where the magnetic energy dominates the electric field energy of the matter coupling to electric charge in the universe. This places particular constraints upon the nature of the cold dark matter dominating the universe today. From our studies in earlier chapters we know that this case provides a slow variation with α increasing logarithmically in time during the dust era but staying constant during any subsequent curvature or cosmological constant dominated era. Also, in a universe with a matter-radiation balance like our own, α remains constant during the radiation era except close to the initial singularity. Negative ζ models are well behaved and correspond to the dark matter in the universe being dominated by magnetic coupling, (for example superconducting cosmic strings contribute $\zeta = -1$). The expansion scale factor evolution is not affected by variations in α to leading order. By contrast, the choice $\zeta > 0$ creates major changes to cosmological evolution. It does not lead to slow increase of α with time during the dust era, as observations suggest, and the evolution of the expansion scale factor is affected to leading order (see for example refs.[6, 7] who discuss related theories for the variation of α with $\zeta > 0$ and hence $\dot{\alpha} < 0$ cosmological behaviour in the dust era in contrast to our discussions in chapters 3-4 and below). In what follows we shall investigate how the $\zeta < 0$ evolution of the fine structure constant couples to variation of G in the Brans-Dicke theory.

The constant ω_{BD} is the Brans-Dicke parameter and ω is the analogous parameter for the coupling of the ψ field driving variations in α. Present observational limits on ω_{BD} are $\omega_{BD} \gtrsim 10^3 - 10^4$. We have used the facts that dust is pressureless, $p = \rho_r/3$ for a sea of radiation and $p = \rho_\psi$ for a fluid with kinetic energy only. Equation (8.3) can be recast for numerical solutions

$$\frac{\dot{a}}{a} = -\frac{1}{2}\frac{\dot{\phi}}{\phi} \pm \frac{1}{2}\sqrt{\left(\frac{\dot{\phi}}{\phi}\right)^2 + \frac{4}{3}\left(\frac{8\pi\rho}{\phi} + \frac{\omega_{BD}}{2}\left(\frac{\dot{\phi}}{\phi}\right)^2\right) - 4\frac{k}{a^2}} \tag{8.9}$$

and eqn. (8.7) integrates to give $\rho_\gamma \exp(-2\psi) \propto a^{-4}$. Note that this is the combination that appears in the generalised Friedmann equation, (8.3). In chapter 4 we showed how to deduce the solutions of these equations when G is constant. Here, we will extend this analysis to the new situation where both α and G vary in time.

8.3 Dust era evolution

From our study of the Friedmann models in BSBM theory we know that, to a very good approximation, the α variations do not significantly affect the evolution of the expansion scale factor $a(t)$. The effects of varying G in Brans-Dicke theories is different. No matter how slow the variation in G, a correction will occur to the power of the time-variation of the expansion scale factor. In the dust era we assume the asymptotic solution for the Brans-Dicke (BD) flat dust model holds to high accuracy. This is an exact solution of (8.3) for $\zeta = k = \psi = \rho_\gamma = 0$ and is the late-time attractor of the general flat BD dust solution (see refs. [8, 9], [10]) which differs only as $t \to 0$, where the solution becomes dominated by the kinetic energy of the ϕ field and approaches the BD vacuum solution. Thus, to leading order the expansion dynamics and ϕ evolution are described at late times by the exact Brans-Dicke dust solution with $k = 0$:

$$a(t) \propto t^{(2-n)/3}; \phi = \phi_0 t^n \tag{8.10}$$

$$\rho = Ma^{-3} \qquad \dot{M} \equiv 0 \tag{8.11}$$

$$\phi_0 \equiv \frac{8\pi M}{n(3+2\omega_{BD})} \tag{8.12}$$

where n is related to the Brans-Dicke parameter by

$$n \equiv 2/(4+3\omega_{BD}), \tag{8.13}$$

and M is the present density of the universe in Planck units, $M \sim 10^{-123}$.

What is the asymptotic solution for α during the dust era? The relevant equation is (8.5), which can now be rewritten as

$$\frac{d}{dt}(t^{2-n}\dot{\psi}) = N\exp(-2\psi) \tag{8.14}$$

where

$$N \equiv -\frac{2\zeta}{\omega}\rho_m a^3 = -\frac{2\zeta M}{\omega} > 0 \qquad \dot{N} \equiv 0,$$

and $-\zeta/\omega \approx 10^{-4}$ is the best fit of this parameter ratio to the observations of Webb et. al. [3]-[5][1] .

Unlike the case with constant G, there is an exact solution (for ω_{BD} positive and finite)

$$\psi(t) = \frac{n}{2}\ln(t) + \frac{1}{2}\ln N - \frac{1}{2}\ln(\frac{n}{2} - \frac{n^2}{2}) \tag{8.15}$$

[1] The value used for $-\zeta/\omega$ is the value fitted for the BSBM theory with constant G. However, since n is so close to zero it should not be significantly different numerically in the case with varying G

so we have, using this solution for ψ to solve for ϕ in (8.4):

$$\alpha(t) = \alpha_0 \exp(2\psi) \;\; = \alpha_0 \frac{2Nt^n}{n(1-n)} \tag{8.16}$$

Hence, there is a simple relationship between $\alpha(t)$ and $G(t)$:

$$\phi = G^{-1} = \frac{2\pi\omega(1-n)}{-\zeta(3+2\omega_{BD})}\frac{\alpha}{\alpha_0} = \frac{2\pi\omega(2+3\omega_{BD})}{-\zeta(3+2\omega_{BD})(4+3\omega_{BD})}\frac{\alpha}{\alpha_0} \propto t^n, \tag{8.17}$$

so αG is always a constant. Note that for large values of ω_{BD} we have a simple relation between the values of G and α:

$$G\frac{\alpha}{\alpha_0} \approx \frac{-\zeta\omega_{BD}}{\pi\omega} > 0 \tag{8.18}$$

As expected, α increases whilst G falls as $t \to \infty$ in a flat universe. It is interesting to note that the asymptotic value of $G\alpha$ is uniquely determined by the parameters in the model with no arbitrary constants.

Although the asymptotic behaviour is now determined, the question of whether this can be reached on a cosmological timescale depends strongly on the choice of initial conditions and needs to be investigated numerically. We can quickly conclude that the asymptotic regime is not reached in our universe. Presently we have $\alpha \approx 1/137$, and in our units the numerical value of M is extremely small, $\sim 10^{-123}$. Obviously the actual value of α is then many orders of magnitude larger than given by the solution in eq.(8.15) and we are thus nowhere near the asymptotic regime. Consequently, in order to find the behaviour of α and G we turn to numerical solutions of the equations. We evolve the Friedmann equations through time with initial conditions chosen so as to yield the present day values of G and α. For α we find a behaviour very similar to the BSBM theory, with a slow growth giving a relative change of the order 10^{-4} throughout the dust epoch. G goes through a decrease of order 10^{-3} during the same period. The numerical results are shown in Figures 8.1 and 8.2.

8.4 Radiation era evolution

The evolution in the radiation era is slightly more complicated because of the contribution of the ρ_ψ term to the right-hand side of the ϕ evolution equation. This means that we do not have the usual late-time asymptotic behaviour of constant ϕ to accompany the $a = t^{1/2}$ scale factor as in BD radiation universes. If we assume

$$a = t^{1/2}$$

then we have

$$\frac{d}{dt}(\dot{\phi}t^{3/2}) = R - \lambda t^{3/2}\dot{\psi}^2 \tag{8.19}$$

where

$$\lambda \equiv \frac{8\pi\omega}{3 + 2\omega_{BD}}, \tag{8.20}$$

$$R \equiv \frac{8\pi M}{3 + 2\omega_{BD}} = -\frac{4\pi\omega N}{\zeta(3 + 2\omega_{BD})} \tag{8.21}$$

The R term is negligible when the kinetic energy of the ψ field dominates the matter density during the radiation era. Likewise, the λ term can be neglected when the matter density dominates the ψ kinetic energy. We also have

$$\frac{d}{dt}(\dot{\psi}t^{3/2}) = N\exp[-2\psi] \tag{8.22}$$

as in the case with constant G. This has the exact solution

$$\psi = \frac{1}{2}\ln(8N) + \frac{1}{4}\ln(t) \tag{8.23}$$

as before, so $\dot{\psi}^2 = (16t^2)^{-1}$. If we substitute this in (8.19)

$$\frac{d}{dt}(\dot{\phi}t^{3/2}) = R - \frac{\lambda}{16}t^{-1/2} \tag{8.24}$$

so

$$\phi = \phi_0 + 2Rt^{1/2} - \frac{\lambda}{8}\ln(t) + Ct^{-1/2} \tag{8.25}$$

where C and ϕ_0 are constants. If the universe expands for long enough to reach the asymptotic regime then we have (as $R > 0$)

$$\psi \approx \frac{1}{4}\ln(t) \tag{8.26}$$

$$\phi = G^{-1} \approx -\frac{8\pi\omega N}{\zeta(3+2\omega_{BD})}t^{1/2} \tag{8.27}$$

so, from eqns. (8.18) and (8.8),

$$G\frac{\alpha}{\alpha_0} \approx \frac{-\zeta(3+2\omega_{BD})}{8\pi\omega N} \approx \frac{-\zeta\omega_{BD}}{4\pi\omega N} \tag{8.28}$$

for large ω_{BD}. Thus we still have the nice asymptotic behaviour of αG in the radiation-dominated epoch. However, we again need to compare with numerical results to determine whether these asymptotic solutions can indeed be realised in the Universe. As in the case of dust, the same simple reality check can now be performed on the solution (8.23). As in the case of constant G we are nowhere near this particular solution in our Universe. α would need to be several orders of magnitude smaller if it was to satisfy the solution, and as in the BSBM theory we expect instead a constant value of α in the rad epoch. This assumption is indeed confirmed by the numerical solutions shown in Figures (8.2) and (8.1).

Another possible problem for the analytic solutions above would arise if the kinetic energy of the ψ field dominates the matter density during radiation domination. We regard this situation as unrealistic and it cannot be realised asymptotically.

8.5 Curvature era evolution

During a curvature-dominated phase of an open universe the expansion scale factor tends to that of the Milne vacuum universe, which is an exact solution of general relativity and of Brans-Dicke theory (with constant ϕ) with

$$a = t \tag{8.29}$$

Using this in the propagation equations for ϕ and ψ,we find that, as $t \to \infty$, so leading order

$$\begin{aligned} \psi &= \psi_* - \frac{N \exp[-2\psi_0]}{t} \\ \phi &= \phi_* + \frac{4\pi\omega N}{\zeta(3+2\omega_{BD})t} \end{aligned}$$

with ψ_* and ϕ_* constants, so both α and G tend to constant values as $t \to \infty$. In a universe that passes directly from dust domination to curvature domination these constant values will be very close to the asymptotic attractors for the dust era of evolution found above in eqn. (8.18) providing the dust epoch has lasted long enough for the attractor to be reached.

The behaviour of G, α and $G\alpha$ in a universe like our own but which eventually becomes dominated by negative curvature is shown in Figure (8.2).

8.6 Cosmological 'constant' era evolution

In flat Brans-Dicke cosmologies a solution of the Friedmann equation with cosmic vacuum energy ($p_v = -\rho_v$) is

$$a = t^{\omega_{BD}+\frac{1}{2}} \tag{8.30}$$

$$\begin{aligned}\phi &= \phi_o t^2 && (8.31)\\ \phi_0 &\equiv \frac{32\pi\rho_v}{(5+6\omega_{BD})(3+2\omega_{BD})} && (8.32)\end{aligned}$$

and ρ_v is constant. This is not the general solution but it is the attractor for the general $p_v = -\rho_v$ solution at late times [11]. It is a power-law inflation model [12]. Note that in Brans-Dicke theory, unlike in general relativity, a $p_v = -\rho_v$ stress behaves differently in the Friedmann equation to an explicit constant Λ term[11]. It is the former that describes the stress contributed by a stationary scalar field with a constant potential. Every term in the BD Friedmann equations falls as t^{-2} for this solution. It is unusual in that it appears to predict that if the universe has just begun accelerating (as observations imply, [13, 14]) then G should vary rapidly in the solar system. However, this argument assumes that the vacuum stress is dominant everywhere, right down to the solar system scale, which in reality it is not.

If we substitute this solution for $a(t)$ (but not ϕ) in the ψ and ϕ evolution equations, (8.4) and (8.5) then we get, since

$$p_v = -\rho_v = const,$$

that

$$\begin{aligned}\ddot{\phi} + \frac{3(2\omega_{BD}+1\ )}{2t}\dot{\phi} &= \frac{8\pi}{3+2\omega_{BD}}(4\rho_v - 2\rho_\psi) \approx \frac{-8\pi}{3+2\omega_{BD}}(4\rho_v - \omega\dot{\psi}^2)\\ \ddot{\psi} + \frac{3(2\omega_{BD}+1\ )}{2t}\dot{\psi} &= -\frac{2}{\omega}e^{-2\psi}\zeta\rho_m \approx 0\end{aligned}$$

So, at late times

$$\begin{aligned}\dot{\psi} &= Aa^{-3} = Dt^{-3(\omega_{BD}+\frac{1}{2})}\\ \psi &= Et^{-3\omega_{BD}-\frac{1}{2}} + F \to F\end{aligned}$$

$$\ddot{\phi} + \frac{3(2\omega_{BD}+1)}{2t}\dot{\phi} = \frac{-8\pi\omega}{3+2\omega_{BD}}(4\rho_v - Qa^{-6}) \to \frac{-32\pi\omega\rho_v}{3+2\omega_{BD}},$$

so

$$\phi = A + Bt^2 + Ct^{-3\omega_{BD}-\frac{1}{2}} \to \phi_0 t^2, \tag{8.33}$$

and, as expected, we get the same growing behaviour as in pure BD. When the universe becomes vacuum-energy dominated α tends to a constant value but $\alpha G \propto G \propto t^{-2}$ continues to fall. This behaviour is confirmed by numerical solutions shown in Figure (8.1). Using eqn. (8.18), we see that if $t_v$is the time when a vacuum-dominated era succeeds a sufficiently long dust-dominated era in a flat universe, then at $t \geq t_v$ in the vacuum-dominated era we expect

$$\alpha(t)G(t) = \frac{-\zeta\omega_{BD}}{\pi\omega}\left(\frac{t_v}{t}\right)^2. \tag{8.34}$$

Hence, today, we would have

$$\alpha(t_0)G(t_0) = \frac{-\zeta\omega_{BD}}{\pi\omega(1+z_v)^{\frac{4}{1+2\omega_{BD}}}}. \tag{8.35}$$

We see that, as in the situation where G is constant, the effect of a vacuum energy or quintessence field is to turn off variations in α when it takes over the expansion of the universe.

8.7 Discussion

We have formulated a simple gravity theory which extends general relativity, by the addition of two scalar fields, to include time variation of G and α. Previously, the study of the cosmological variation of physical 'constants' has confined attention to varying one constant only or to discussing the effects of altering the values of physical constants without a self-consistent theory for their dynamical variation [15], [16]. The structure of unified gauge theories and particle physics theories with extra dimensions has given some indication as to the self consistency conditions required if traditional constants are allowed to vary, [17, 18].

We have found that the expansion of the universe is affected by varying G to first order and the evolution of the expansion scale factor follows the behaviour found in Brans-Dicke cosmologies to leading order without being significantly affected by variations in α. The variations in α are affected by the variations in G through their influence on the expansion rate. This is significant in the dust-dominated era of cosmic expansion, which in chapter 4 was shown to exhibit a special mathematical behaviour in the absence of G variation. The effect of any G time variation simplifies the α variation and allows an exact solution to be found with $\alpha \propto t^n$,where $n \equiv 2/(4+3\omega_{BD})$ is determined by the Brans-Dicke parameter ω_{BD}.

In both the radiation and dust dominated eras, there are asymptotic solutions in which the product $G\alpha$ remains constant and its value is determined uniquely by the coupling constants of the theory. However in universes like our own with the values of α and G near present values these asymptotic regimes are not reached throughout the life of the universe. Typically our present values for α are much larger than the values required by the asymptotic solution.

In a curvature-dominated or quintessence-dominated era the variation in α ceases, just as in the situation with no G variation investigated in chapters 3 and 4. This is an important feature of all models with varying α in theories of the BSBM sort because it naturally reconciles evidence of variations in α at redshifts $z \sim 1-3$, with local ($z = 0.1$) constraints from the Oklo natural reactor if the universal expansion began to accelerate at $z \sim 0.7$, as current observations imply.

Finally, we reiterate that the conclusions drawn above apply only to varying-α

theories with negative ζ. The exact solutions given in eqs. (8.15),(8.16),and (8.23) for the evolution of $\alpha(t)$ during the radiation and dust eras no longer exist when $\zeta > 0$ and hence $N < 0$. During the curvature and cosmological constant-dominated eras the evolution of $a(t)$ is significantly changed by the variations of ψ and the assumptions (8.29) and (8.30) for the scale factor evolution are no longer valid.

The study performed here provides a simple cosmological model in which the variation of two 'constants' can be studied exactly. A number of extensions are possible. The variations of weak and strong couplings can be included and the constraints imposed by any scheme grand unification can be imposed [17].

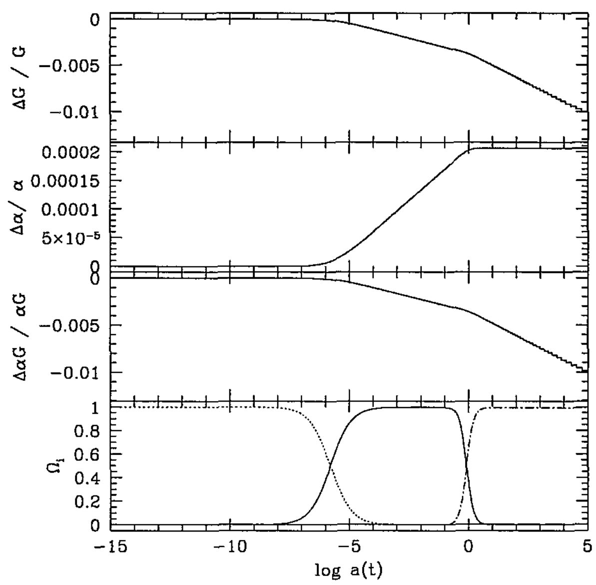

Figure 8.1: Evolution of the relative shift in the values of the two 'constants' in a realistic flat cosmology with vacuum energy and approximately accurate initial values for the fields. We start from a radiation-dominated universe where both α and G stay constant. Thereafter we move into dust domination where α changes slowly, while G goes through a small decrease. As the universe becomes dominated by the vacuum energy, α goes to a constant, while G goes on decreasing indefinitely as in ordinary Brans Dicke theory. Values used for the couplings are the minimum allowed value of $\omega_{BD} = 3500$ and we take the best fit value of $\zeta/\omega = -10^{-4}$ from BSBM theory.

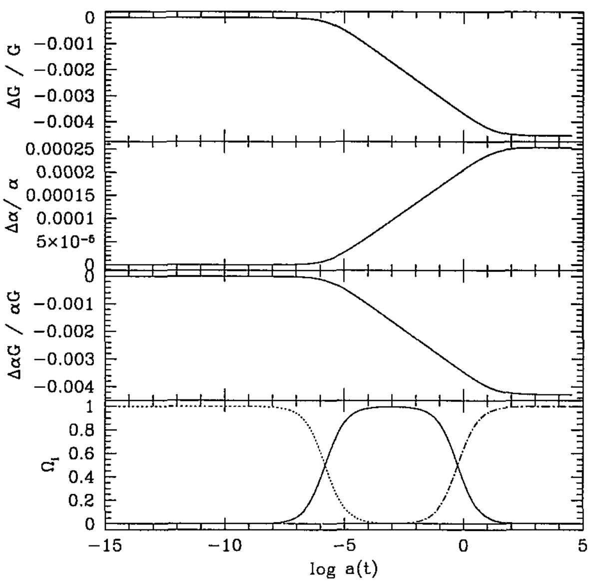

Figure 8.2: Evolution of the relative shift in the values of the two 'constants' in an open universe through radiation, dust and curvature-dominated epochs. Initial values for the fields are set so as to give realistic values at present time. Again α and G are constants in the radiation dominated era, whilst α increases and G decreases through matter domination. As curvature starts to take over the expansion, both α and G tends to constants. Values for the couplings are the minimum allowed value of $\omega_{BD} = 3500$ and the best fit value of $\zeta/\omega = -10^{-4}$ from the BSBM theory.

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Chapter 9

Conclusions and Outlook

The results by Webb et. al. [1, 2, 3] suggesting a time-varying fine structure constant have sparked new intensity into the debate about the value and constancy of physical 'constants'. Partially motivated by these results, the main theme of this thesis has been the development and subsequent investigation of a generalisation of Bekenstein's theory. In this theory the dimensionless fine structure constant $\alpha = e^2/(\hbar c)$ is allowed to vary through the presence of a dielectric field pervading the vacuum. The variation is thus effectively contributed to the dimensionful electric charge e.

We started by considering soliton solutions to Bekenstein's original theory, and found vortex solutions (cosmic strings) around which the electric charge is varying due to the magnetic energy in the string core. Typically the value of the electric charge is much higher near the core of the string. We discussed cosmological consequences of networks of such strings, particularly the possibility of inhomogeneous recombination scenarios.

From here we went on to generalise Bekenstein's theory to include the self gravitation of the scalar field carrying the α variation (BSBM theory). We showed how we can account for the variation found by Webb et. al. and how by putting constraints on the nature of dark matter we still marginally honour constraints from Etwös type tests of the universality of free fall. The behaviour of this theory in the various cosmological epochs was thoroughly investigated through exhausting analytical and numerical studies. Maybe the most important feature of the model is that there is only a very short window in the dust-epoch in which α changes. Hence we are able to honour simultaneously both early universe constraints from Big Bang Nucleosynthesis[4, 5] and geonuclear constraints[6, 7] as well as the Webb results. Then inhomogeneous cosmological variations in α in this theory were considered, and we found that inhomogeneities are wiped away with time and solutions approach the behaviour found in exact Friedmann universes.

Later we discussed some novel anthropic considerations that arise from the realisation in earlier chapters that α will vary as long as the Universe is dominated by pressureless dust. This places lower bounds on the cosmological constant and on curvature if life is to be sustainable in the Universe. Similar anthropic arguments were found for Brans-Dicke varying-G theory. This complements earlier anthropic arguments for upper limits on these quantities.

We also looked at ways in which different varying-α theories can be distinguished by their different impacts on the weak equivalence principle (WEP). Our own BSBM theory has far stronger WEP violations than e.g. varying speed of light theories where the scalar field couples universally to the **whole** matter lagrangian. Thus the improved precision of several orders of magnitude from experiments already in the planning stages should be enough to determine which of e or c is varying.

Since there is no theoretical reason why a potential variation in the physical 'constants' should be confined to just one constant, we finally laid out the framework for a theory allowing for a variation in both α and the gravitational constant G. The relevant Friedmann equations were derived and the cosmological behaviour was analysed.

There are several possible avenues to explore within the BSBM framework. One interesting question is what consequences our theory might have for high energy phenomena like pulsars and black holes. We have developed the classical solutions for a charged black hole in this theory (see appendix B), but it is important to investigate what impact this might have on other aspects of black holes (for instance along the lines of what is suggested by [8]). It is also interesting to try to link our theory with low-energy limits of more fundamental theories in want of a more solid theoretical foundation.

There are genuine problems in trying to fit the quasar data[1, 2, 3] with theoretical models[9, 10, 11] as well as other constraints on α. Banks et. al. [12] have showed how even tiny variations in alpha could pose major problems for renormalization in Quantum Field Theory. In the BSBM theory, maybe the most conservative varying-α theory, these problems have been highlighted by the need for 'unnatural' dark matter in order to comply with fifth-force constraints. However, with space based tests of the weak equivalence principle planned in the near future[13] there is hope that some of these models can be ruled out or confirmed. A point of caution is that due to the immense importance of the quasar results, it is vital they be independently verified and extended. Should they be confirmed, no doubt most of the physical constructions employed by physicists will have to be reexamined.

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Appendix A

Friedmann equations

Below we present a more rigorous transition from the Einstein equations to the Friedmann equations. Especially the details of the ζ parameter for electric and magnetic energy densities will be examined.

Again the Einstein equations are,

$$G_{\mu\nu} = 8\pi G \left(T^{mat}_{\mu\nu} + T^{\psi}_{\mu\nu} + T^{em}_{\mu\nu} e^{-2\psi} \right), \qquad \text{(A.1)}$$

with the dynamical equation for the scalar field:

$$\Box\psi = \frac{2}{\omega} e^{-2\psi} \mathcal{L}_{em} \qquad \text{(A.2)}$$

We need to introduce the ζ parameters in order to parameterise the fraction of electric and magnetic energies:

$$\zeta^E_m = \frac{E^2}{\rho_m}, \qquad \zeta^B_m = \frac{E^2}{\rho_m} \qquad \text{(A.3)}$$

where ρ_m is the density of non-relativistic matter in the Universe, and E^2 and B^2 are the electric and magnetic energies respectively. Thus equation A.3 can be written

$$\Box\psi = \frac{2}{\omega} e^{-2\psi} \rho_m (\zeta^E_m - \zeta^B_m), \qquad \text{(A.4)}$$

and assuming isotropy and homogeneity we obtain the new version of equation 3.5

$$\ddot{\psi} + 3H\dot{\psi} = -\frac{2}{\omega} e^{-2\psi} \rho_m (\zeta^E_m - \zeta^B_m). \qquad \text{(A.5)}$$

Since the time-time component of the electromagnetic stress-energy tensor is

$$T^{00}_{em} = \frac{1}{2}\left(E^2 + B^2\right) \tag{A.6}$$

we get the the **sum** of the two ζ parameters in the Friedmann equation

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3}\left(\rho_m\left(1 - (\zeta^E_m + \zeta^B_m) + (\zeta^E_m + \zeta^B_m)e^{-2\psi}\right) + \rho_r e^{-2\psi} + \rho_\psi\right) + \frac{\Lambda}{3}. \tag{A.7}$$

where we have employed a FRW metric. This is a more rigorous variant of eqn. (3.4), which can be obtained by defining $\zeta_m = \zeta^E_m - \zeta^B_m$, introducing absolute values in eqn. (A.7) and taking the limit where either $\zeta^E_m \gg \zeta^B_m$ (positive ζ_m) or $\zeta^E_m \ll \zeta^B_m$ (negative ζ_m). Eqn. (3.4) only becomes significantly inaccurate for the very special case where $\zeta^E_m \sim \zeta^B_m$, and we have chosen to use the simplified form throughout this thesis.

Appendix B

Black Hole Solutions in the BSBM theory

B.1 Introduction

One of the most interesting aspects of varying α theories are their implications for black holes and stars. In order to test varying alpha theories experimentally, we need to know their impact on stars and black holes on astrophysical scales. Varying c theories are known[4] to result in a divergent speed of light on the horizon. We will show that in the framework of BSBM - theory of a varying fundamental electric charge, the charge is indeed well behaved on the horizon. We will show that this theory has solutions **identical** to dilatonic black holes carrying electric charge.

B.2 Action and Equations of Motion

The relevant action where the vacuum is endowed with only the electromagnetic field is

$$S = -\int d^4x\sqrt{-g}\left(\frac{1}{16\pi G}R - \frac{1}{4}e^{-2\psi}F^2 - \frac{\omega}{2}\partial_\mu\psi\partial^\mu\psi\right) \tag{B.1}$$

This is similar to the action used for the dilatonic black hole described in [2, 3], which reads

$$S = -\int d^x\sqrt{-g}\left(\frac{1}{16\pi G}R - \frac{1}{4}e^{-2a\phi}F^2 - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi\right) \tag{B.2}$$

Thus the theories are identical with $\psi = a\phi$ and $\omega = 1/a^2$.

The Einstein equations are obtained from varying the action with respect to the metric,

$$G_{\mu\nu} = -8\pi G T_{\mu\nu} \tag{B.3}$$

and from variation with respect to the gauge field a_μ and the scalar field we obtain the modified Maxwell equations and the dynamical equation for ψ respectively,

$$\partial_\nu \frac{\sqrt{g} f^{\mu\nu}}{\epsilon^2} = 0 \tag{B.4}$$

$$\frac{1}{\sqrt{g}} \partial_\mu \left(\sqrt{g} \partial^\mu \psi\right) = -\frac{e^{-2\psi}}{2\omega} f_{\mu\nu} f^{\mu\nu} \tag{B.5}$$

B.3 Solutions

The solution of the field equations are remarkably simple and can be easily obtained by using the results in refs. [2, 3] together with the redefinitions from the previous section. The metric is

$$ds^2 = -\lambda^2 dt^2 + \frac{dr^2}{\lambda^2} + R^2 d\Omega \tag{B.6}$$

where $\lambda^2 = (1 - \frac{r_+}{r})(1 - \frac{r_-}{r})^{\frac{\omega-1}{\omega+1}}$ and $R = r(1 - \frac{r_-}{r})^{\frac{1}{1+\omega}}$. For the gauge field we have

$$f_{tr} = \frac{Q}{r^2}, \tag{B.7}$$

and for the ψ field:

$$e^{\psi} = \left(1 - \frac{r_-}{r}\right)^{\frac{1}{1+\omega}} \tag{B.8}$$

or equivalently

$$\psi = \frac{1}{1+\omega} ln(1 - \frac{r_-}{r}). \tag{B.9}$$

The radii r_+ and r_- are related by

$$M = \frac{r_+}{2} + \left(\frac{\omega - 1}{\omega + 1}\right) \frac{r_-}{2} \tag{B.10}$$

$$Q = \sqrt{\frac{\omega}{\omega + 1} r_+ r_-} \tag{B.11}$$

There are several interesting aspects to this solution. Firstly note that there are two horizons. For $r = r_+$ we have a non-singular event horizon as for the normal Schwartzchild solution. The inner horizon is singular for all finite values of ω. Only in the no coupling limit $\omega \to \infty$ do we recover the Reissner-Nordström solutionwith two non-singular horizons.

It is also interesting to note that the solution for ψ (and thus the electric charge e) is **finite** and well behaved at the horizon $r = r_+$ with $\psi = 1/(1+\omega)\ln(1 - r_-/r_+)$. This is to be contrasted with the solutions found for VSL theories in [4] for which the speed of light either diverges or goes to zero on the horizon.

B.4 Discussion

For minimally coupled scalar fields, no-hair theorems for black holes apply[5, 6] for both the massive and massless cases and for any kind of potential. In other words the black hole cannot possess any quantum number associated with these fields. The same is true for massive vector fields. The massless vector field, has an internal gauge freedom which makes the black hole horizon non pathological. This property also allows a scalar field to escape the theorem if it is strongly coupled to the electromagnetic field, as is the case in the BSBM theory and dilaton theories. We found that the BSBM solution is identical to the solution for the charged dilaton black hole, and the electromagnetic coupling is indeed well behaved and finite on the event horizon. We leave it to future work to further investigate the details and consequences of this solution.

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